Supplementary information to "Evolution of Conditional and Unconditional Commitment"

02/09/20

Contents

1 Introduction

We analyze an evolutionary-game-theoretic model of coordination under uncertainty. In a first section we introduce the basic game, and analyze the simple case where agents cannot communicate with each other. Then we analyze the game where players can use signals.

An agent's strategy profile is defined by continuous variables. It is convenient to adopt the following notation to refer to them. A variable is denoted by a simple greek letter (e.g. β); the value of this variable in a population playing a given strategy is denoted $\bar{\beta}$, while the equilibrium value of the variable is β^*.

2 Basic model (without communication)

The basic game is a variant of the Stag Hunt with private information. It has two players. First, each player i draws a number from a uniform distribution $[0, 1]$, this number represents the payoff S_i of successful coordination to that player in the game. The value of S_i is private information and cannot be communicated to the other player. Then players play the game represented by the following matrix:

Table 1: Payoff matrix

	Stay	Walk
Stay	S_i, S_j	$0, w$
Walk	$w, 0$	w, w

where the value of w is similar for both players and is common knowledge.

Just as in a normal Stag Hunt, Walking away gives a riskless payoff, that is always the same regardless of the partner's move. Staying is potentially more profitable, but is risky, because it yields payoff 0 if the partner Walks.

2.1 ESS of the basic game

We first analyze this basic game when there is no opportunity for communication between players. Notably, we show that when $w > \frac{1}{4}$, there is only one equilibrium, where players always Walk.

If there exists an ESS where Staying occurs with positive probability, then this ESS must be a threshold strategy of the form "Stay if $S_i > \beta$, Walk otherwise", where $\beta \in [0, 1]$. To see why, let $p > 0$ be the probability that the other player will Stay. Regardless of whether the other player plays a threshold strategy, the payoff-maximizing strategy is to Stay if $pS_i > w$, and Walk away if $pS_i < w$. This is the threshold strategy just described above. (Note that agents with *high* values of β are *less* likely to Stay. For instance, at the extremes, an agent with $\beta = 1$ always Walks, while an agent with $\beta = 0$ always Stays.)

In a population where the average value of β is $\bar{\beta}$, the optimal strategy for an agent i is to stay if:

$$(1 - \bar{\beta})S_i > w$$

because the payoff of Staying is equal to S_i discounted by the probability that the other agent is staying. S is drawn from a uniform distribution, so there is probability $1 - \bar{\beta}$ that the other agent will Stay.

This is equivalent to:

$$S_i > \frac{w}{1-\bar{\beta}}$$

β is precisely the threshold for S_i above which an agent Stays, so this equation can be read as saying that the best reply to $\bar{\beta}$ is given by the recursion function:

$$R(\beta) = \frac{w}{1-\bar{\beta}}$$

At equilibrium, β must be a best reply to itself:

$$\beta^* = \frac{w}{1-\beta^*}$$

Solving the quadratic equation yields:

$$\beta^* = .5 \pm \frac{\sqrt{1-4w}}{2}$$

In section 4.1, we show that only the equilibrium defined by the lower root (i.e., $\beta^* = .5 - \frac{\sqrt{1-4w}}{2}$) is of biological significance, because the upper root equilibrium is not continuously stable (after an arbitrarily small perturbation to the system away from that equilibrium, the population will be pushed ever further away from it). Since $\beta \in [0,1]$, this implies that as long as $w > 0$, then $\beta^* = 1$, i.e. "Always Walk", is an equilibrium, just as Hare Hunting is an equilibrium in a standard Stag Hunt game. Intuitively, in a population where noone ever Stays, it cannot be rational to Stay, therefore strategies that sometimes Stay cannot invade.

Note that for an equilibrium other than "Always Walk" to exist, it must be the case that $1 - 4w > 0$, i.e. $w > \frac{1}{4}$. Otherwise, the best reply to $\bar{\beta}$ is always greater than $\bar{\beta}$, and the population settles to $\bar{\beta} = 1$, where no player ever stays. When outside options are too great, there is a runaway escalation of distrust, stifling coordination.

2.2 Private information makes coordination more difficult

Note that in an equivalent Stag Hunt with complete information, there exists an equilibrium where everyone always Stays, even when $w > \frac{1}{4}$. For instance the following game has an ESS where every player always stays, even though the average payoffs (for succesful coordination and for Walking Away) are the same as in the private information game with $w = .3$:

Table 2: Payoff matrix

	Stay	Walk
Stay	$.5, .5$	$0, .3$
Walk	$.3, 0$	$.3, .3$

2.3 The paradox of outside options

At equilibrium, the expected payoff of an agent can be computed as:

$$E(\beta^*) = (1 - \beta^*)^2 S_i + \beta^* w$$

where $S = \frac{1+\beta^*}{2}$ is the average payoff of an agent who successfully coordinates with its partner. There is a probability $(1 - \beta^*)^2$ that both the agent and his partner stay, yielding S_i to the agent, and a probability β^* that the agent walks and get a payoff of w. Numerical computations show that in the interval

$[0, .25]$, an agent's expected payoff is a *decreasing* function of w: better outside options make agents worse off on average (see figure below).

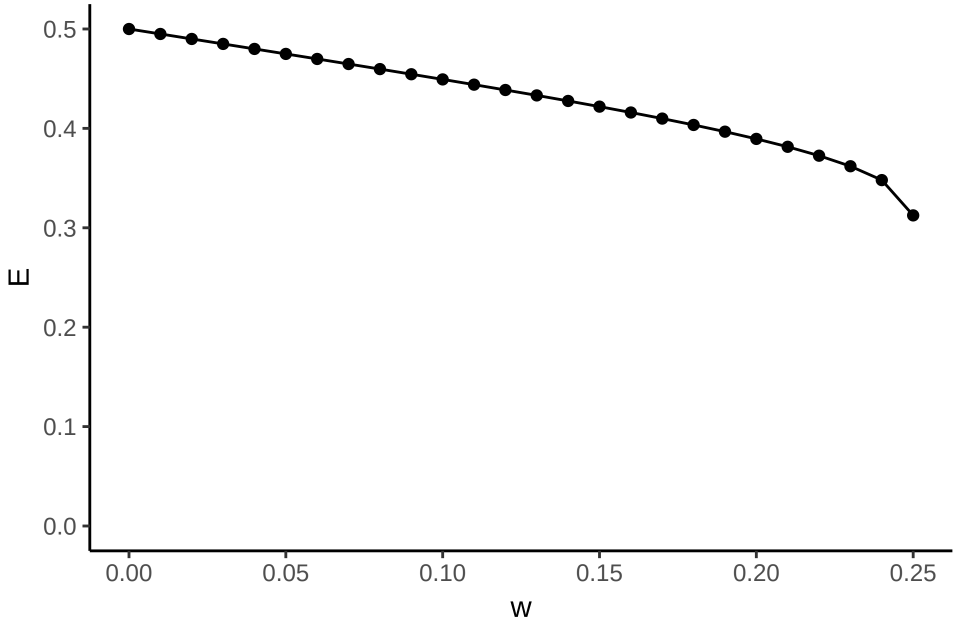

Figure S1: **Expected payoff as a function of w**

3 Introducing Signaling

The basic game just described is preceded by a signaling stage in which both players, after observing their value of S, are allowed to pay a cost $c > 0$ in order to send a signal. Players take the decision whether or not to Signal at the same time. Then players are allowed to condition their behavior in the coordination game to whether they have received a signal.

We assume that a player's strategy in this extended game is a triplet (α, β, γ). α defines the value of S_i above which the player sends a signal. β defines the value of S_i above which the player Stays in the absence of a signal from the other player; γ represents the same value in the presence of a signal. We focus on pure strategies.

We will say of an agent who has drawn S_i such that $\gamma < S_i < \beta$ that he *Stays conditionally*, to refer to the fact that he Stays only if he receives a signal. An agent with $\beta < S_i$ *Stays unconditionally*, that is, regardless of whether he receives a signal.

3.1 Signaling equilibria

Let us first consider signaling equilibria. A signaling equilibrium of this game is an ESS with $\alpha^* < 1$, that is, an ESS where players sometimes Signal.

The fact that $c > 0$ ensures that there are only negative consequences to Signaling when one intends to Walk Away. When an agent intends to Walk, he does not get any benefits from Signaling, because his payoff does not depend on what his partner does. By Signaling, such an agent would only pay c without receiving any compensating benefits. This ensures that natural selection is stacked against deceptive signalers. Only agents who intend to Stay can possibly get benefits from Signaling, which makes the signal honest. The interesting question is then: can there be equilibria where the benefits of Signaling are sufficient to outweigh its cost?

It is easy to see how a signaling ESS must have $\beta^* > \gamma^*$ (i.e., agents are more likely to Stay when they receive a signal), otherwise Signaling has no benefits. Note also that α^* cannot be smaller than both β^* and γ^*: an agent playing this strategy would sometimes Signal when he strictly does not intend to Stay, which is suboptimal.

There are four different types of strategies that obey these constraints: strategies with $\alpha > \beta > \gamma$; $\beta > \alpha > \gamma$; $\alpha = \gamma$; or $\alpha = \beta$. As we will show, there exist Evolutionarily Stable Strategies (ESS) for each of these four types. However, the strategy space in this game is continuous (genes can take any value between 0 and 1), and in a continuous space being an ESS is not always significant. This is because some ESS are *continuously unstable*: if a slight perturbation takes the population even slightly away from that ESS, then evolutionary dynamics will take the population ever further away from the ESS. Therefore we are mostly interested in the more restrictive concept of a Continuously Stable Strategy (CSS). CSS are ESS that are continuously stable - populations will come back to the CSS even after a slight perturbation of the system.

We will show that only strategies which obey either $\alpha = \gamma$ or $\alpha = \beta$ can be CSS. For each strategy type, we derive the conditions under which such a strategy can be ESS, then we determine whether they are continuously stable. The latter proofs are slightly more technical than the rest, so we present them in separate sections toward the end of this document.

3.1.1 Case where $\alpha = \gamma$

Note that a signaling equilibrium with $\alpha^* = \gamma^*$ must also have $\beta^* = 1$. We need $\beta^* > \gamma^*$ for signaling to be of any benefit. If an agent j does not Signal, it means that this agent's S_j is lower than α^*, which also means that S_j has to be lower than both γ^* and β^*. Therefore agents who do not Signal never Stay, so one should always Walk away from a non-signaling partner.

Intuitively a strategy with $\alpha = \gamma$ and $\beta = 1$ is a strategy where a signal means "I will Stay if and only if you have also sent me a signal".

We first establish that if a strategy with $\alpha = \gamma$ cannot be invaded by a mutant who deviates from the resident strategy in both α and γ simultaneously, then the strategy cannot be invaded by any mutant and is therefore ESS. The intuition is that agents with $\alpha = \gamma$ are more "internally consistent" than agents with $\alpha \neq \gamma$: in a population with $\bar{\alpha} = \bar{\gamma}$, it is irrational for an agent to have unequal values for the two variables. If "internally consistent" agents cannot invade, neither can "internally inconsistent" agents - because they have lower fitness. The goal is to show that, for purposes of equilibrium analysis, we can treat α and γ as if they were the same variable.

Claim. Let us call "inconsistent" a mutant with $\alpha \neq \gamma$, and "consistent" a mutant with $\alpha = \gamma$. Then, for any inconsistent mutant, there exists a consistent mutant with a better expected payoff.

Proof. There are two possible types of inconsistent mutants:

-Mutant with $\alpha < \gamma$. There is an interval in which the mutant Signals even though he will not Stay (when the mutant draws S_i such that $\alpha < S_i < \gamma$). This is irrational, and a consistent mutant with the same value of γ, but with $\alpha = \gamma$ performs better because he avoids such wasteful signaling.

-Mutant with $\gamma < \alpha$. There is an interval in which the mutant Stays even though he did not Signal (when the mutant draws S_i such that $\alpha > S_i > \gamma$). This is irrational, because no resident Stays in the absence of a signal from their partner (since $\beta^* = 1$). Therefore a consistent mutant with the same value of α but $\gamma = \alpha$ performs better because he pockets the outside option in situations where the inconsistent mutant Stays in vain.

Therefore, the $\alpha = \gamma$ strategy is ESS if and only if it can resist invasion by a consistent mutant. As a result, only the latter type of mutant is relevant to equilibrium analysis. Not having to worry about inconsistent mutants means that we can treat α and γ as if they were the same variable in the following analysis.

Value of α^*. To compute the equilibrium value α^* (which is identical to γ^*), we compare the expected payoff of an agent who Signals and Stays conditionally (denoted $E(S)$) to that of an agent who does not Signal and does not Stay (denoted $E(\neg S)$). We have:

$$E(S) = \alpha^*(w - c) + (1 - \alpha^*)(S_i - c)$$

(with probability α^*, the partner does not Signal, so the agent Walks away and gets w; with probability $1 - \alpha^*$, the partner Signals and Stays, so coordination is successful and the agent gets S_i. In both cases, the agent pays the cost c of Signaling)

and:

$$E(\neg S) = w$$

An agent should Signal and Stay conditionally when:

$$E(S) > E(\neg S)$$

→

$$\alpha^*(w - c) + (1 - \alpha^*)(S_i - c) > w$$

→

$$S_i > \frac{c + w - \alpha^* w}{1 - \alpha^*}$$

Therefore, α^* must satisfy:

$$\alpha^* = \frac{c + w - \alpha^* w}{1 - \alpha^*}$$

Solving the quadratic equation yields:

$$\alpha^* = \frac{1 + w}{2} \pm \frac{\sqrt{(1 - w)^2 - 4c}}{2}$$

In section 4.2, we show that only the equilibrium defined by the lower root is a CSS.

Existence conditions. Note that these equilibria only exist if $(1 - w)^2 - 4c > 0$, that is if $w < 1 - 2\sqrt{c}$. When $w > 1 - 2\sqrt{c}$, runaway escalation of distrust makes coordination impossible, and selection takes the population to $\bar{\alpha} = 1$.

The intuition behind the result is the following. If communication was both free and honest, then agents would Signal and Stay conditionally whenever they have $S_i > w$, because there is no risk or cost in doing so, and α^* $(= \gamma^*)$ should be w. With a cost to the signal, then one requirement for signaling to be worthwhile is that $S_i - c > w$. But an agent knows that his partner will not Signal unless she herself has $S_j - c > w$, meaning that he must discount the potential payoff of successful coordination by the probability that his partner will Signal. As a consequence, the agent's threshold for Signalling / Staying conditionally should be a little larger than $w + c$. Furthermore, the partner knows that the agent anticipates this, which drives her estimate of the likelihood that the agent will stay even lower, and so on. When neither w nor c are too high, this process converges to a value of α^* between 0 and 1, but when $w > 1 - 2\sqrt{c}$, then the process results in no signaling.

Invasion potential. As discussed further in section 3.2.3, this strategy cannot formally invade a population of non-signalers, but its large basin of attraction makes it likely to prevail nonetheless, for values of w and c low enough, if chance plays a sufficienly large role in evolutionary dynamics (e.g. high mutation rate, small population).

Case of very small costs. Note that, as remarked above, as c tends toward 0, α^* and γ^* tend toward w. In the idealized setting of the analytic model, even a very small cost to signaling is enough to make silence dominant over signaling for agents who intend to Walk Away, therefore it makes the signal honest. Thus, a very small c approximates the situation where signaling is both free and honest. Intuitively, when signals are truthful yet very inexpensive, an agent should communicate that his incentives favor coordination whenever this is the case. Furthermore, as c tends toward 0, the value of w above which the signaling equilibrium is unstable tends towards 1, i.e. the equilibrium tends toward always existing no matter how high the outside options. Of course, infinitesimally small costs are not biologically realistic. Also, because chance usually plays a role in evolutionary processes, very small costs will not actually constitute a sufficient selection pressure against fake signals. Because they incorporate the role of chance, agent-based simulations (see main text) provide a more realistic treatment of this case.

3.1.2 Case where $\alpha = \beta$

Intuitively, such a strategy is one of *unconditional commitment*, where the signal means "I will Stay, not matter whether you Signaled or not".

Value of γ^*. The fact that $\alpha^* = \beta^*$ implies that a signaling agent always Stays, regardless of whether he receives a signal. Therefore, an agent who receives a signal is certain that his partner will Stay, and should Stay if and only if $S_i > w$. Therefore, at equilibrium, $\gamma^* = w$.

Then, the general approach is the same as for the $\alpha = \gamma$ case. We first show that a strategy that resists invasion by consistent mutants is ESS.

Claim. Let us call "inconsistent" a mutant with $\alpha \neq \beta$, and "consistent" a mutant with $\alpha = \beta$. Then, for any inconsistent mutant, there exists a consistent mutant with a higher expected payoff.

Proof. There are two possible types of inconsistent mutants:

-Mutant with $\alpha < \beta$. There exists an interval (when $\alpha < S_i < \beta$) where the mutant Signals, but does not Stay if he does not receive a signal. In a population that plays $\bar{\alpha} = \bar{\beta}$, this is irrational. If the partner Signals, she also

Stays unconditionally, so there is no need to send her a signal. If the partner does not Signal, the mutant does not Stay (as per $S_i < \beta$), so he does not get the benefits of having sent the signal. In either case, Signaling is costly but has no benefits. A consistent mutant, with the same value of β but $\alpha = \beta$ avoids wasteful signaling.

-Mutant with $\beta < \alpha$. There exists an interval (when $\beta < S_i < \alpha$) where the mutant Stays unconditionally, but does not Signal. In this interval, he would actually be better off Staying conditionally: since he has not Signaled, the partner will only Stay if she has $S_j > \bar{\beta}$, i.e. if she has $S_j > \bar{\alpha}$. That is, if the partner does not Signal she also does not Stay. A consistent mutant, with the same value of α but with $\beta = \alpha$, Stays conditionally in this case, and pockets the outside option more often.

Therefore, a strategy that can resist invasion by consistent mutants can resist invasion by any mutant, and is an ESS. As a result, only the latter type of mutant is relevant to equilibrium analysis. Not having to worry about inconsistent mutants means that we can treat α and β as if they were the same variable in the following analysis.

To compute the equilibrium value α^* (which is identical to β^*), we compare the expected payoff of an agent who Signals and Stays unconditionally (denoted $E(S)$) to that of an agent who does not Signal and Stays only in the presence of a signal (denoted $E(\neg S)$). We have:

$$E(S) = (1 - w)S_i - c$$

$$E(\neg S) = (1 - \alpha)S_i + \alpha w$$

(In the first case, the partner Stays with probability $1 - w$, and you pay the cost c of Signaling. In the second case, the partner Stays with probability $1 - \beta (= 1 - \alpha)$; and Walks with probability α; it is possible to detect that she will Walk from the absence of a signal, and pocket the outside option in this case).

An agent should Signal and Stay unconditionally when:

$$E(S) > E(\neg S)$$

$\rightarrow$

$$(1 - w)S_i - c > (1 - \alpha)S_i + \alpha w$$

$\rightarrow$

$$S_i > \frac{c + \alpha w}{\alpha - w}$$

Therefore α^* must verify:

$$\alpha^* = \frac{c + \alpha^* w}{\alpha^* - w}$$

Solving the quadratic equation yields:

$$\alpha^* = w + \sqrt{w^2 + c}$$

Therefore, the strategy is a signaling ESS whenever $w + \sqrt{w^2 + c} < 1$, i.e., whenever $w < \frac{1 - c}{2}$. We show in section 3.2.1 that it can invade a population of non-signalers, and in section 4.3 that it is also a CSS.

3.1.3 Case where $\alpha > \beta > \gamma$

We look for an equilibrium strategy $(\alpha^*, \beta^*, \gamma^*)$, such that $\alpha^* > \beta^* > \gamma^*$.

Value of γ^*. The assumption that $\alpha^* > \beta^*$ implies that a signaling agent always Stays, regardless of whether he receives a signal. Therefore, an agent who receives a signal is certain that his partner will Stay, and should Stay if and only if $S_i > w$. Therefore, at equilibrium, $\gamma^* = w$.

Value of β^*. We are looking for the threshold above which an agent starts Staying even in the absence of a signal from his partner. By assumption, this threshold is below α^* but above γ^*. Therefore, we look for the value of S_i that makes an agent indifferent between Staying conditionally and unconditionally, when this agent has not Signaled. To do so, we consider an agent who has neither given not received a signal, and ask when he is better off Staying or Walking. This agent can infer that his partner will Stay with probability $\frac{\alpha^*-\beta^*}{\alpha^*}$: the fact that the partner has not Signaled means her value of S_j is below α^*, and there is therefore a probability $\frac{\alpha^*-\beta^*}{\alpha^*}$ that her value of S_j is above β^* (which she needs in order to Stay). Consequently, the agent should Stay only if:

$$\frac{\alpha^* - \beta^*}{\alpha^*} S_i > w$$

meaning that:

$$\beta^* = \frac{\alpha^* w}{\alpha^* - \beta^*}$$

Value of α^*. We are looking for the threshold above which an agent Signals. By assumption, this threshold is greater than β^*. Therefore, we look for the value of S_i that makes an agent indifferent between Signaling and staying silent, when this agent intends to Stay unconditionally. This agent should Signal if the benefit of signaling is greater than its cost. The benefit of signaling is simply S_i times the increase in the likelihood that the partner Stays, and its cost is c. The partner Stays with probability $1 - w$

when receiving a signal, and with probability $1 - \beta^*$ otherwise, so Signaling increases the likelihood of the partner Staying by $\beta^* - w$. Therefore, the agent Signals if:

$$S_i(\beta^* - w) > c$$

meaning that:

$$\alpha^* = \frac{c}{\beta^* - w}$$

Values of α^* and β^* as a function of w and c. The equation derived above for β^* can be rearranged to yield:

$$\beta^*(\alpha^* - \beta^*) = \alpha^* w$$

so that

$$(\beta^*)^2 = \alpha^*(\beta^* - w)$$

while the equation for α^* can be rewritten as

$$\alpha^*(\beta^* - w) = c$$

Combining the two facts yields

$$\beta^* = \sqrt{c}$$

We can then substitute β^* by $\sqrt{c}$ in the equation for α^*

$$\alpha^* = \frac{c}{\sqrt{c} - w}$$

However, we show in section 4.4 that this equilibrium is not continuously stable.

3.1.4 Case where $\beta > \alpha > \gamma$

We look for an equilibrium strategy $(\alpha^*, \beta^*, \gamma^*)$ such that $\beta^* > \alpha^* > \gamma^*$.

Value of γ^*. By assumption, at equilibrium we have $\gamma^* < \alpha^*$, meaning that an agent who is indifferent between Staying and Walking away when receiving a signal does not himself Signal. Therefore, we consider an agent who has not Signaled but has received a signal from his partner, and ask when he is better off Staying or Walking. In the absence of a signal, the partner will only Stay if she has $S_j > \beta^*$. Therefore she Stays with probability $\frac{1-\beta^*}{1-\alpha^*}$. Then the agent's expected benefit for Staying is $\frac{1-\beta^*}{1-\alpha^*}S_i$. γ^* is the value of S_i that makes the agent indifferent between Staying and Walking away:

$$\frac{1-\beta^*}{1-\alpha^*}\gamma^* = w$$

i.e.

$$\gamma^* = \frac{1-\alpha^*}{1-\beta^*}w$$

Value of β^*. By assumption, at equilibrium $\beta^* > \alpha^*$, meaning that an agent who is indifferent between Staying and Walking in the absence of a signal is the kind of agent who himself has sent a signal. Therefore, we consider an agent who has Signaled but has not received a signal, and ask when he is better off Staying or Walking. The partner has not Signalled, so she must have $S_j < \alpha^*$, so she Stays with probability $\frac{\alpha^*-\gamma^*}{\alpha^*}$. Then the agent's expected benefit for Staying is $\frac{\alpha^*-\gamma^*}{\alpha^*}S_i$. β^* is the value of S_i that makes the agent indifferent between Staying and Walking away:

$$\frac{\alpha^*-\gamma^*}{\alpha^*}\beta^* = w$$

i.e.

$$\beta^* = \frac{\alpha^*}{\alpha^* - \gamma^*} w$$

Value of α^*. By assumption, at equilibrium we have $\gamma^* < \alpha^* < \beta^*$, meaning that an agent who is indifferent between Signaling and staying silent also intends to Stay conditionally. Therefore we consider an agent who intends to Stay conditionally, and ask whether he is better off Signaling or not. For such an agent, Signaling only turns out to be useful when his partner has $\beta > S_j > \alpha$: if $S_j > \beta$, she would have Stayed even in the absence of a signal; if $S_j < \alpha$, she does not Signal, prompting the agent to Walk. The partner has $\beta > S_j > \alpha$ with probability $\beta - \alpha$. In this interval, Signaling yields the agent a payoff S_i, while staying silent prompts the partner to Walk, yielding no payoff. Taking into account the cost of the signal, the expected payoff of Signaling is therefore:

$$E(Signal) = (\beta^* - \alpha^*)S_i - c$$

At equilibrium, the agent is indifferent between Signaling and staying silent:

$$E(Signal) = 0$$

i.e.

$$\alpha^* = \frac{c}{\beta^* - \alpha^*}$$

However, we show in section 4.5 that no equilibrium satisfying these conditions is continuously stable.

3.2 Non-signaling equilibria

A non-signaling equilibrium is an equilibrium with $\alpha^* = 1$. Note that such equilibria are not formally ESS, but rather neutral equilibria, because γ is allowed to drift. This is due to the fact that since nobody Signals, there is no selection pressure on γ.

Whether and when non-signaling strategies are equilibria is interesting, if only because it tells us whether and when signaling strategies can invade. There are two possible classes of non-signaling strategies, depending on whether β is equal to or lower than 1.

3.2.1 Case where $\alpha = \beta = 1$

Agents in a population playing this strategy never Signal and never Stay. It cannot be invaded by a mutant who deviates only in $\alpha < 1$, because such a mutant, having still $\beta = 1$ only Stays when receiving a signal - and noone in the population ever Signals. This mutant would then always Signal vacuously. Therefore we ask whether such a population would be invaded by a mutant who deviates in both α and β. Specifically, we focus on the case of mutants with $\alpha = \beta$, because for any mutant with $\alpha \neq \beta$, there exists a player with $\alpha = \beta$ who does better (an agent with $\beta > \alpha$ sometimes signals even though he will not Stay; an agent with $\alpha > \beta$ sometimes Stays without signaling, meaning he Stays even though it is certain that his partner will not). We also assume that the mutant plays $\gamma = w$, which is the optimal value of γ when the mutant meets itself.

A necessary requirement for such a mutant to do better than the resident is for drift to have taken the value of $\bar{\gamma}$ to a sufficiently low value, such that signaling has a non-trivial effect on the likelihood that a member of the resident population is going to Stay when receiving a signal. In theory, drift could take $\bar{\gamma}$ to 0, but then $\bar{\gamma}$ would increase to w as soon as signaling re-appears, w being the optimal value for γ given that $\bar{\alpha} = \bar{\beta}$. Therefore to find out whether the non-signaling strategy can be subject to an invasion with long-lasting effects, we look at whether selection could bring $\bar{\alpha}$ down given that $\bar{\gamma} = w$.

In a population with $\bar{\alpha} = 1$; $\bar{\beta} = 1$; $\bar{\gamma} = w$, the optimal strategy is to Signal (and then Stay unconditionally) if and only if:

$$(1 - w)S_i - c > w$$

where the left-hand side is the expected payoff for signaling (and then Staying unconditionally), and the right-hand side the expected payoff for not signaling (and then Walking away). For $S_i = 1 - \epsilon$, this is equivalent to:

$$(1 - w)(1 - \epsilon) - c > w$$

i.e.

$$1 - \epsilon - 2w + \epsilon w - c > 0$$

Taking the limit of the left-hand side as ϵ approaches 0, this is equivalent to

$$1 - 2w - c > 0$$

i.e.

$$w < \frac{1-c}{2}$$

If $w < \frac{1-c}{2}$, then there exists a signaling strategy with $\alpha = \beta = 1 - \epsilon$ that can invade the non-signaling strategy. Therefore, a strategy with $\alpha = \beta = 1$ is stable against invasion by a signaling strategy only for $w \geq \frac{1-c}{2}$. As we show in 3.1.3 and 4.3, there is only one signaling CSS with $\alpha^* = \beta^*$; furthermore, this CSS is stable precisely when $w < \frac{1-c}{2}$. Therefore, if the non-signaling strategy is invaded by a signaling strategy, then evolutionary dynamics will take the population to the signaling CSS with $\alpha^* = \beta^*$.

3.2.2 Case where $\alpha = 1$, $\beta < 1$

The selection pressure on $\bar{\beta}$ in this population is equivalent to the one in the no-communication version of the game, since no signals are emitted here. Therefore, one condition for the strategy to be an equilibrium is that $w < \frac{1}{4}$, otherwise selection will take $\bar{\beta}$ to 1.

Let us assume that $w < \frac{1}{4}$, such that $\bar{\beta}$ is maintained below 1 (specifically, at $.5 - \frac{\sqrt{1-4w}}{2}$, which corresponds to β^* in the no-communication-game). In such a population, when can selection bring $\bar{\alpha}$ down? A necessary requirement for $\bar{\alpha}$ to decrease is for drift to have taken the value of $\bar{\gamma}$ at least below $\bar{\beta}$ (otherwise signaling has no benefits). In theory, drift could take $\bar{\gamma}$ to 0, but then $\bar{\gamma}$ would increase to w as soon as signaling re-appears, w being the optimal value for γ given that $\bar{\alpha} > \bar{\beta}$. Therefore to find out whether the non-signaling strategy can be subject to an invasion with long-lasting effects, we look at whether selection could bring $\bar{\alpha}$ down given $\bar{\gamma} = w$.

Let us consider an agent with $S_i \approx 1$. $E(Signal, Stay)$ is the payoff to that agent if he Signals and Stays unconditionally, while $E(\neg Signal, Stay)$ is the payoff to an agent who Stays unconditionally but does not Signal.

Signaling can invade when

$$E(Signal, Stay) > E(\neg Signal, Stay)$$

$$(1-w)S_i - c > (1-\bar{\beta})S_i$$

i.e

$$1 - w - c > 1 - \bar{\beta}$$

i.e.

$$\bar{\beta} - w > c$$

Substituting $.5 - \frac{\sqrt{1-4w}}{2}$ for $\bar{\beta}$, this can be rewritten as

$$c < \frac{1 - 2w - \sqrt{1-4w}}{2}$$

For instance, if $w = .24$, then Signaling can invade if $c < .16$. In the interval $[0, \frac{1}{4}]$, the value of $\frac{1-2w-\sqrt{1-4w}}{2}$, and thus the benefits to Signaling, increase as a function of w. Therefore, when w is very small, the benefits of Signaling are negligible, and even small values of c can prevent Signaling from invading. As w increases, Signaling can more easily invade, even when relatively costly. Intuitively, small outside options mean that agents can trust that their partner will Stay most of the time, making Signaling superfluous. When outside options are sufficiently large that they may jeopardize coordination, Signaling becomes useful, and invades more easily.

3.2.3 Signaling can invade silence and promote coordination where it would otherwise not be possible

In a situation where $w > \frac{1}{4}$, coordination is not possible without communication. In such a situation, can an initially silent population ever be invaded by a signaling strategy? As just shown above (in 3.2.1), such a population, playing the strategy ($\alpha = 1$, $\beta = 1$) can be invaded by the $\alpha^* = \beta^*$ signaling strategy under the same conditions that make the latter strategy stable. This shows that costly signaling can invade and foster coordination even in an initially silent population. The signaling strategy $\alpha^* = \gamma^*$ cannot formally invade such a silent population. However, if stochastic shocks bring the value of $\bar{\alpha}$ and $\bar{\gamma}$ of a silent population within the basin of attraction of the signaling strategy, then the population will be pushed to that signaling ESS. This is a likely possibility, since the strategy has a large basin of attraction - its upper boundary is equal to $\alpha^+ = \frac{1+w}{2} + \frac{\sqrt{(1-w)^2-4c}}{2}$. To give an example, for $w = .3$ and $c = .05$, the upper boundary value of the basin of attraction is .92, meaning that if stochastic shocks take $\bar{\alpha}$ and $\bar{\gamma}$ below .92, the population will be attracted to the signaling equilibrium.

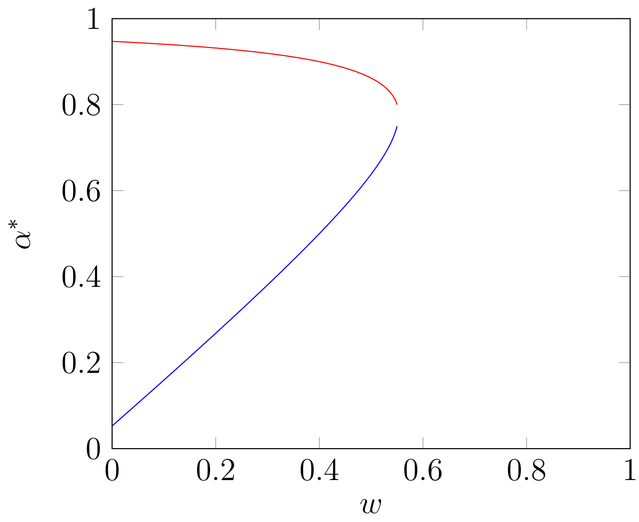

Figure S2: **Values of the $\alpha^* = \gamma^*$ equilibria for each value of w.** The red line plots unstable equilibria, while the blue line plots stable equilibria. c is fixed at $c = .05$. Any population whose values of $\bar{\alpha}$ and $\bar{\gamma}$ lie between the red line and the lower abcissa will be attracted to the blue line

4 Continuous Stability

4.1 Basic game (without communication)

Recall from section 2.1 that the basic game has the following two equilibria:

$$\beta^* = .5 \pm \frac{\sqrt{1-4w}}{2}$$

We now show that only the equilibrium defined by the lower root is continuously stable, meaning that the population will come back to that equilibrium after a small perturbation only for the lower root.

Let us assume that an equilibrium exists, meaning that $w < \frac{1}{4}$. Let R' be the derivative of the recursion function with respect to β. For an equilibrium

β^*, if $0 < R'(\beta^*) < 1$, then the equilibrium is stable, and if $R'(\beta^*) > 1$, then the equilibrium is unstable (e.g., Bulmer & Parker, 2002). The basic idea is that if $R'(\beta^*) > 1$ then if by a perturbation to the system, β increases by a small amount δ above the equilibrium, then the best reply to β increases by more than δ, i.e. there is positive feedback away from the equilibrium.

As a reminder, the function giving the best reply to β is:

$$R(\beta) = \frac{w}{1-\bar{\beta}}$$

Its derivative is given by:

$$R'(\beta) = \frac{w}{(1-\bar{\beta})^2}$$

First, we observe that

$$R'(.5) = \frac{w}{(1-.5)^2} = 4w$$

We know that $w < \frac{1}{4}$. Therefore $R'(.5) < 1$. The equilibrium defined by the lower root is $\beta^- = .5 - \frac{\sqrt{1-4w}}{2} < .5$.The fact that $R'(\beta)$ is monotonically increasing in $\bar{\beta}$ implies then that:

$$R'(\beta^-) < R'(.5)$$

$$R'(\beta^-) < 1$$

Therefore the lower equilibrium is stable. Together with the fact that $\lim_{\beta\to 1} R(\beta) = \infty$, this implies, by a simple graphical argument, that $R'(\beta^+) > 1$, and therefore the upper root equilibrium must be unstable:

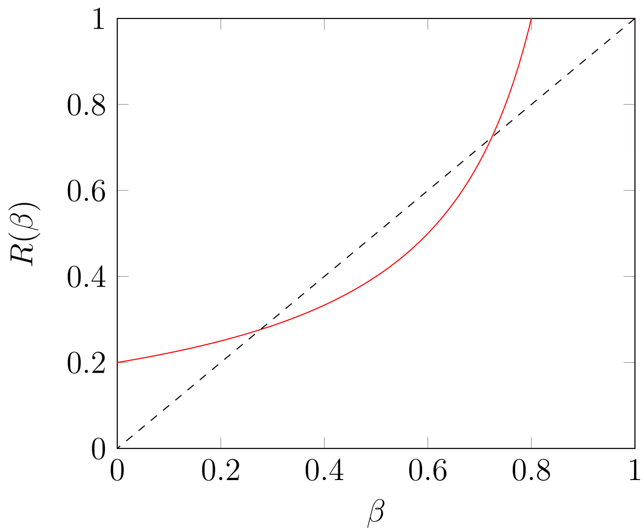

Figure S3: (The equilibria are at the intersection between the recursion function (red line) and the identity line.)

4.2 Equilibrium with $\alpha^* = \gamma^*$

Recall that in section 3.1.1 we found the following two equilibria, defined by:

$$\alpha^* = \gamma^* = \frac{1+w}{2} \pm \frac{\sqrt{(1-w)^2 - 4c}}{2}$$

We now show that only the equilibrium defined by the lower root is continuously stable.

First we establish that we can treat α and γ as if they were the same variable for the purposes of continuous stability analysis.

Claim. In the vicinity of the equilibrium, evolutionary dynamics always take the population to a state where $\bar{\alpha} = \bar{\gamma}$.

Proof. There are two possible situations:

→ $\bar{\alpha} > \bar{\gamma}$: In this situation, there is a small interval where agents do not Signal but Stay conditionally nonetheless. Since $\bar{\beta} = 1$, partners never Stay in the absence of a signal, therefore Staying without Signaling is irrational. Agents should either Signal more often, or Stay less often, in any case, the population is taken to $\bar{\alpha} = \bar{\gamma}$.[1]

→ $\bar{\alpha} < \bar{\gamma}$: In this situation, there is a small interval where agents Signal but do not Stay. This is irrational, and agents should either Signal less often or Stay more often. In any case, the population is taken to $\bar{\alpha} = \bar{\gamma}$.

As a consequence, a population slightly out-of-equilibrium always converges to a state where $\bar{\alpha} = \bar{\gamma}$. We now ask whether such a population is attracted toward the equilibrium, or pushed away from it. $R_\alpha(\alpha)$ is the function which gives the best reply value of α (which by necessity is also the best reply value of γ) in a population where $\bar{\alpha} = \bar{\gamma}$. Note that the optimal value of β is always 1 in the vicinity of the equilibrium, so β is not important to the stability analysis.

We denote the lower and the upper equilibria as $\alpha^- = \frac{1+w}{2} - \frac{\sqrt{(1-w)^2-4c}}{2}$ and $\alpha^+ = \frac{1+w}{2} + \frac{\sqrt{(1-w)^2-4c}}{2}$.

---

[1] The assumption that $\bar{\beta} = 1$ may seem no longer warranted when $\bar{\alpha} > \bar{\gamma}$, because it is not possible anymore to assume that a partner who does not Signal will never Stay. Yet the optimal value of β is still 1, for the following reason. The probability that an agent who does not Signal will Stay conditionally is $\frac{\bar{\alpha}-\bar{\gamma}}{\bar{\alpha}}$, which by hypothesis is very small, so even an agent with $S_i = 1$ should Walk away if his partner did not Signal (because $w > \frac{\bar{\alpha}-\bar{\gamma}}{\bar{\alpha}}1$)

From section 3.1.1, the best reply function for α is:

$$R_\alpha(\alpha) = \frac{c + w - \alpha w}{1 - \alpha}$$

Its derivative with respect to α is:

$$R'_\alpha(\alpha) = \frac{c}{(1-\alpha)^2}$$

For an equilibrium α^*, if $0 < R'_\alpha(\alpha^*) < 1$ then α^* is stable, and if $R'_\alpha(\alpha^*) > 1$ it is unstable. Note that $R'_\alpha(\alpha^*) < 1 \iff 1 - \alpha^* > \sqrt{c}$. We first show that α^- is stable.

$$1 - \alpha^- = \frac{1-w}{2} + \frac{\sqrt{(1-w)^2 - 4c}}{2}$$

We know that $w < 1 - 2\sqrt{c}$, otherwise the equilibrium does not exist. This is equivalent to:

$$\frac{1-w}{2} > \sqrt{c}$$

Together with $\frac{\sqrt{(1-w)^2-4c}}{2} > 0$, this implies that:

$$1 - \alpha^- > \sqrt{c}$$

$$\iff R'_\alpha(\alpha^-) < 1$$

meaning α^- is stable.

Since $\lim_{\alpha \to 1} R(\alpha) = \infty$, a simple graphical argument shows that $R'_\alpha(\alpha^+) > 1$, and the upper equilibrium is unstable:

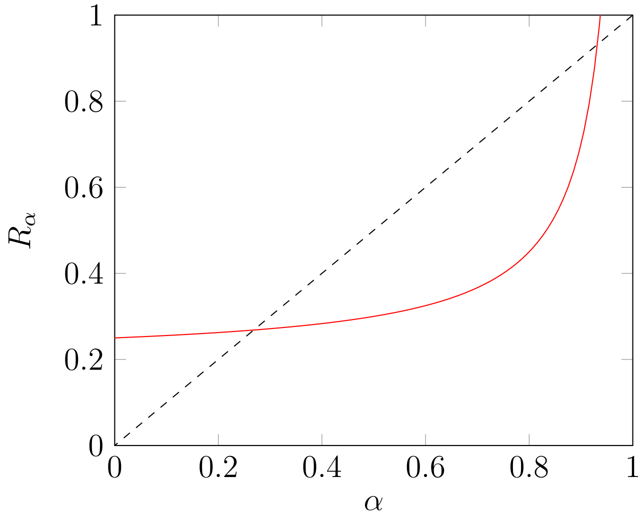

Figure S4: (The equilibria are at the intersection between the recursion function (red line) and the identity line.)

4.3 Equilibrium with $\alpha^* = \beta^*$

First we establish that we can treat α and β as if they were the same variable for the purposes of continuous stability analysis. Note that the optimal value of γ is always w in the vicinity of the equilibrium, so the variable is not important for stability analysis.

Claim. In the vicinity of the equilibrium, evolutionary dynamics always take the population to a state where $\bar{\alpha} = \bar{\beta}$.

Proof. There are two possible situations:

$\rightarrow \bar{\alpha} > \bar{\beta}$. Let $\epsilon > 0$ be the (very small) difference between $\bar{\alpha}$ and $\bar{\beta}$, such that $\bar{\alpha} = \bar{\beta} + \epsilon$. There exists an interval (when $\bar{\alpha} > S_i > \bar{\beta}$) where agents

Stay unconditionally, but do not Signal. But in this interval, you are actually better off Staying conditionally (i.e. you should Walk if you do not receive a signal). Here is why. Assume you have drawn S_i such that $\bar{\alpha} > S_i > \bar{\beta}$; this implies that you did not signal. In the absence of a signal, your partner will Stay only if she draws $S_j > \bar{\beta}$. Assume further that you do not receive a signal from your partner. This means that your partner has drawn $S_j < \bar{\alpha}$. It follows from these two facts that your partner Stays with probability $\frac{\bar{\alpha}-\bar{\beta}}{\alpha}$, i.e. she Stays with probability $\frac{\epsilon}{\alpha}$. In this case, your expected payoff for Staying is $\frac{\epsilon}{\alpha}S_i$, whose limit is 0 as ϵ approaches 0. A mutant with a higher value of β, such that $\beta = \bar{\alpha}$ gets a higher payoff because in this interval he pockets the outside option instead of uselessly Staying.

→ $\bar{\beta} > \bar{\alpha}$. Let $\epsilon > 0$ be the (very small) difference between $\bar{\alpha}$ and $\bar{\beta}$, such that $\bar{\beta} = \bar{\alpha} + \epsilon$. There exists an interval (when $\bar{\beta} > S_i > \bar{\alpha}$) where an agent Signals, and Stays only if he receives a signal. But in this interval, you are actually better off not signaling. This is true regardless of whether your partner will signal or not. First, assume that your partner signals. This means she had $S_j > \bar{\alpha}$; therefore she will Stay with probability 1 if she received a signal, and with probability $\frac{1-\bar{\beta}}{1-\alpha} = \frac{1-(\bar{\alpha}+\epsilon)}{1-\alpha}$ if she did not receive a signal. In this case, signaling only raised the likelihood that your partner will stay by $\frac{\epsilon}{1-\alpha}$, so its marginal benefit is $\frac{\epsilon}{1-\alpha}S_i$, whose limit is 0 as ϵ approaches 0. Second, assume that your partner does not signal. Because you have drawn $S_i < \bar{\beta}$, you will not Stay, so you should not signal. In both cases, refraining from signaling is better. Therefore, a mutant with a higher value of α such that $\alpha = \bar{\beta}$ avoids wasteful signaling and gets a higher payoff.

As a consequence, a population slightly out-of-equilibrium always converges to a state where $\bar{\alpha} = \bar{\beta}$. We now ask whether such a population is attracted toward the equilibrium, or pushed away from it.

The method employed in the last two sections to determine continuous stability does not work as well here because we have $R'(\alpha^*) = -1$, which is not enough information to determine whether the equilibrium is stable or unstable. This method consisted in looking at the best reply to a given strategy - instead, in the following, for a given strategy we look at strategies directly in this strategy's neighborhood and ask whether they have higher fitness (see McNamara et al., 2003).

Let $E(\alpha, \bar{\alpha})$ be the expected payoff of an agent playing α in a population playing $\bar{\alpha}$. Then, let $f(\bar{\alpha})$ be the partial derivative of E with respect to α, evaluated at $\alpha = \bar{\alpha}$. Intuitively, $f(\bar{\alpha})$ gives the direction of selection on α in a population playing $\bar{\alpha}$: e.g. when $f(\bar{\alpha})$ is negative, an agent with a value of α slightly below the population average enjoys a higher expected payoff than an agent playing the resident strategy. We go yet one level deeper and consider the partial derivative of this partial derivative. An equilibrium α^* is continuously stable if:

$$\frac{\partial f}{\partial \bar{\alpha}}\Big|_{\bar{\alpha}=\alpha^*} < 0$$

In other words, if, at equilibrium, a change in the population value $\bar{\alpha}$ in a given direction causes a change in the selection pressure on α in the opposite direction, the equilibrium is stable (see MacNamara et al., 2003). The intuition is that such negative feedback keeps pushing the population back to equilibrium.

Consider a mutant playing $\bar{\alpha}+d\alpha$ (where $d\alpha$ is very small) in a population playing $\bar{\alpha}$. The difference in fitness between mutant and resident is:

$$\Delta E = E(\bar{\alpha}+d\alpha) - E(\bar{\alpha})$$

Their behavior only differs in an interval of size $|d\alpha|$, where one agent stays quiet and Stays conditionally, while the other agent Signals and Stays unconditionally. We denote $E(S)$ the payoff of an agent in this interval who Signals and Stays unconditionally, and $E(\neg S)$ the payoff of an agent in this interval who does not Signal and Stays conditionally. Note that an agent is in this interval when he has S_i between $\bar{\alpha}$ and $\bar{\alpha}+d\alpha$ (i.e. $S_i \approx \bar{\alpha}$). We have:

$$\Delta E = |d\alpha|(E(\neg S) - E(S))$$

if $d\alpha$ is positive, and

$$\Delta E = |d\alpha|(E(S) - E(\neg S))$$

$$= -|d\alpha|(E(\neg S) - E(S))$$

$$= d\alpha(E(\neg S) - E(S))$$

if $d\alpha$ is negative.

So whatever the sign of $d\alpha$ we have

$$\Delta E = d\alpha(E(\neg S) - E(S))$$

From the definition of a derivative, we know that

$$f = \frac{E(\bar{\alpha}+d\alpha) - E(\bar{\alpha})}{d\alpha} = \frac{\Delta E}{d\alpha}$$

$$= \frac{d\alpha(E(\neg S) - E(S))}{d\alpha}$$
$$= E(\neg S) - E(S)$$

From section 3.1.2, we know that $E(S) = (1-w)S_i - c$ and $E(\neg S) = (1-\bar{\alpha})S_i + \bar{\alpha}w$. Then:

$$f = (1-\bar{\alpha})S_i + \bar{\alpha}w - (1-w)S_i + c$$

$$= (w - \bar{\alpha})\bar{\alpha} + \bar{\alpha}w + c$$

$$= -\bar{\alpha}^2 + 2w\bar{\alpha} + c$$

Therefore we have:

$$\frac{\partial f}{\partial \bar{\alpha}}\Big|_{\bar{\alpha}=\alpha^*} = -2\alpha^* + 2w$$
$$\frac{\partial f}{\partial \bar{\alpha}}\Big|_{\bar{\alpha}=\alpha^*} < 0$$

and the equilibrium is stable.

4.4 Equilibrium with $\alpha^* > \beta^* > \gamma^*$

Note that the optimal value of γ is always w in the vicinity of the equilibrium, so the variable is not important for stability analysis.

We use the same technique as in the previous section, except that the stability problem is two-dimensional: the equilibrium can be disrupted if the system is taken away from α^*, β^* or both. Therefore, instead of evaluating the sign of a partial derivative, we examine a matrix of partial derivatives.

Let $W(\beta, \bar{\alpha}, \bar{\beta})$ be the expected payoff of an agent playing β in a population playing $(\bar{\alpha}, \bar{\beta})$, and $f_\beta(\bar{\alpha}, \bar{\beta})$ be the partial derivative of W with respect to β, evaluated at $(\beta = \bar{\beta}, \alpha = \bar{\alpha})$. $f_\beta(\bar{\alpha}, \bar{\beta})$ gives the direction of selection on β in a population playing $(\bar{\alpha}, \bar{\beta})$: e.g. when f_β is negative, an agent with a value of β slightly below the population average enjoys a higher expected payoff than an agent playing the resident strategy. $W(\alpha, \bar{\alpha}, \bar{\beta})$ and $f_\alpha(\bar{\alpha}, \bar{\beta})$ are defined similarly. $f_\alpha(\bar{\alpha}, \bar{\beta})$ and $f_\beta(\bar{\alpha}, \bar{\beta})$ govern the dynamics of the system.

Let A be the Jacobian matrix of the system at equilibrium:

$$A = \begin{bmatrix} \frac{\partial f_\alpha}{\partial \bar{\alpha}} & \frac{\partial f_\alpha}{\partial \bar{\beta}} \\ \frac{\partial f_\beta}{\partial \bar{\alpha}} & \frac{\partial f_\beta}{\partial \bar{\beta}} \end{bmatrix}$$

where all partial derivatives are evaluated at (α^*, β^*).

Then a necessary condition for the equilibrium to be continuously stable is that $tr(A) \leq 0$ and $det(A) \geq 0$ (MacNamara et al., 2003).

Consider a mutant playing $\bar{\beta} + d\beta$ (where $d\beta$ is very small) in a population playing $\bar{\beta}$. The difference in fitness between mutant and resident is:

$$\Delta W = W(\bar{\beta} + d\beta) - W(\bar{\beta})$$

The two agents only differ in behavior in an interval of size $|d\beta|$, where one agent Walks while the other agent Stays. $W(Walk)$ and $W(Stay)$ denote the payoff for Walking or Staying to an agent with $S_i \approx \bar{\beta}$. We have:

$$\Delta W = |d\beta|(W(Walk) - W(Stay))$$

if $d\beta$ is positive, and

$$\Delta W = |d\beta|(W(Stay) - W(Walk))$$

$$= -|d\beta|(W(Walk) - W(Stay))$$

590 if $d\beta$ is negative.

So whatever the sign of $d\beta$ we have

$$\Delta W = d\beta(W(Walk) - W(Stay))$$

By the definition of a derivative, we know that:

$$f_\beta(\bar{\alpha}, \bar{\beta}) = \frac{W(\bar{\beta} + d\beta) - W(\bar{\beta})}{d\beta}$$

$$f_\beta(\bar{\alpha}, \bar{\beta}) = \frac{d\beta(W(Walk) - W(Stay))}{d\beta}$$

$$f_\beta(\bar{\alpha}, \bar{\beta}) = W(Walk) - W(Stay)$$

$W(Stay)$ is given by the probability that one's partner Stays given that no agent has Signaled, times the payoff $S_i = \bar{\beta}$:

$$W(Stay) = \frac{\bar{\alpha} - \bar{\beta}}{\bar{\alpha}}\bar{\beta}$$

while $W(Walk)$ is simply w. Therefore,

$$f_\beta(\bar{\alpha}, \bar{\beta}) = w - \frac{\bar{\alpha} - \bar{\beta}}{\bar{\alpha}}\bar{\beta}$$

591 The same reasoning shows that:

$$f_\alpha(\bar{\alpha}, \bar{\beta}) = \bar{\alpha}(w - \bar{\beta}) + c$$

The Jacobian matrix of the system is then:

$$A = \begin{bmatrix} w - \beta^* & -\alpha^* \\ -(\frac{\beta^*}{\alpha^*})^2 & \frac{2\beta^* - \alpha^*}{\alpha^*} \end{bmatrix}$$

We now show that $det(A) < 0$ (dropping the * for convenience).

$$det(A) = \frac{2\beta - \alpha}{\alpha}(w - \beta) - \alpha(\frac{\beta}{\alpha})^2$$

$$= \frac{2w\beta - 2\beta^2 - \alpha w + \alpha\beta}{\alpha} - \frac{\beta^2}{\alpha}$$

$\alpha > 0$, therefore the sign of $det(A)$ is the sign of

$$2w\beta - \alpha w + \alpha\beta - 3\beta^2$$

Substituting the equilibrium values of α and β (see section 3.1.3), this is equal to:

$$= 2w\sqrt{c} - \frac{wc}{\sqrt{c} - w} + \frac{c\sqrt{c}}{\sqrt{c} - w} - 3\sqrt{c}^2$$

$$= 2w\sqrt{c} - 3c + \frac{c(\sqrt{c} - w)}{\sqrt{c} - w}$$

$$= 2w\sqrt{c} - 2c$$

$$= 2\sqrt{c}(w - \sqrt{c})$$

The fact that $\alpha^* = \frac{c}{\sqrt{c}-w}$ and $\alpha^* > 0$ imply that $\sqrt{c} - w > 0$, i.e. $w - \sqrt{c} < 0$. Therefore,

$$2\sqrt{c}(w - \sqrt{c}) < 0$$

$$det(A) < 0$$

and the equilibrium is continuously unstable.

4.5 Equilibrium with $\beta^* > \alpha^* > \gamma^*$

The basic approach is the same as in the previous section: we use the Jacobian matrix associated with a given equilibrium to determine whether it is stable. Using the same reasoning as before, the values for f_α, f_β, and f_γ are:

$$f_\alpha = W(\neg Signal) - W(Signal)$$

$$= c - (\beta - \alpha)\alpha$$

$$f_\beta = W(Walk) - W(Stay|\neg Signal)$$

$$= w - \frac{\alpha - \gamma}{\alpha}\beta$$

where $|\neg Signal$ means "given that the partner has not Signaled".

$$f_\gamma = W(Walk) - W(Stay|Signal)$$

$$= w - \frac{1-\beta}{1-\alpha}\gamma$$

Then the Jacobian matrix for the system is

$$A = \begin{bmatrix} \frac{\partial f_\alpha}{\partial \bar{\alpha}} & \frac{\partial f_\alpha}{\partial \bar{\beta}} & \frac{\partial f_\alpha}{\partial \bar{\gamma}} \\ \frac{\partial f_\beta}{\partial \bar{\alpha}} & \frac{\partial f_\beta}{\partial \bar{\beta}} & \frac{\partial f_\beta}{\partial \bar{\gamma}} \\ \frac{\partial f_\gamma}{\partial \bar{\alpha}} & \frac{\partial f_\gamma}{\partial \bar{\beta}} & \frac{\partial f_\gamma}{\partial \bar{\gamma}} \end{bmatrix}$$

where all partial derivatives are evaluated at $(\alpha^*, \beta^*, \gamma^*)$:

$$A = \begin{bmatrix} 2\alpha^* - \beta^* & -\alpha^* & 0 \\ -\frac{\beta^*\gamma^*}{\alpha^{*2}} & \frac{\gamma^*-\alpha^*}{\alpha^*} & \frac{\beta^*}{\alpha^*} \\ \frac{\beta^*-1}{(1-\alpha^*)^2}\gamma^* & \frac{\gamma^*}{1-\alpha^*} & -\frac{1-\beta^*}{1-\alpha^*} \end{bmatrix}$$

An equilibrium is unstable if at least one eigenvalue of this matrix is positive; it is stable if all three eigenvalues are negative.

To first derive the equilibria, one can for instance use the following formulas, which produce unique values of β^*, w, and c, given α^* and γ^*.

Combining the facts that:

$$\frac{1-\beta^*}{1-\alpha^*}\gamma^* = w$$

and

$$\frac{\alpha^* - \gamma^*}{\alpha^*}\beta^* = w$$

we have:

$$\beta^* = \frac{\alpha^*\gamma^*}{(1-\alpha^*)(\alpha^* - \gamma^*) + \alpha^*\gamma^*}$$

and we already know (from section 3.1.4) that:

$$w = \frac{\alpha^* - \gamma^*}{\alpha^*}\beta^*$$

$$c = \alpha^*(\beta^* - \alpha^*)$$

Then numerical simulation shows that when an equilibrium exists, the associated Jacobian matrix always has at least one positive eigenvalue, i.e. that equilibrium is unstable.

5 Agent-based simulations

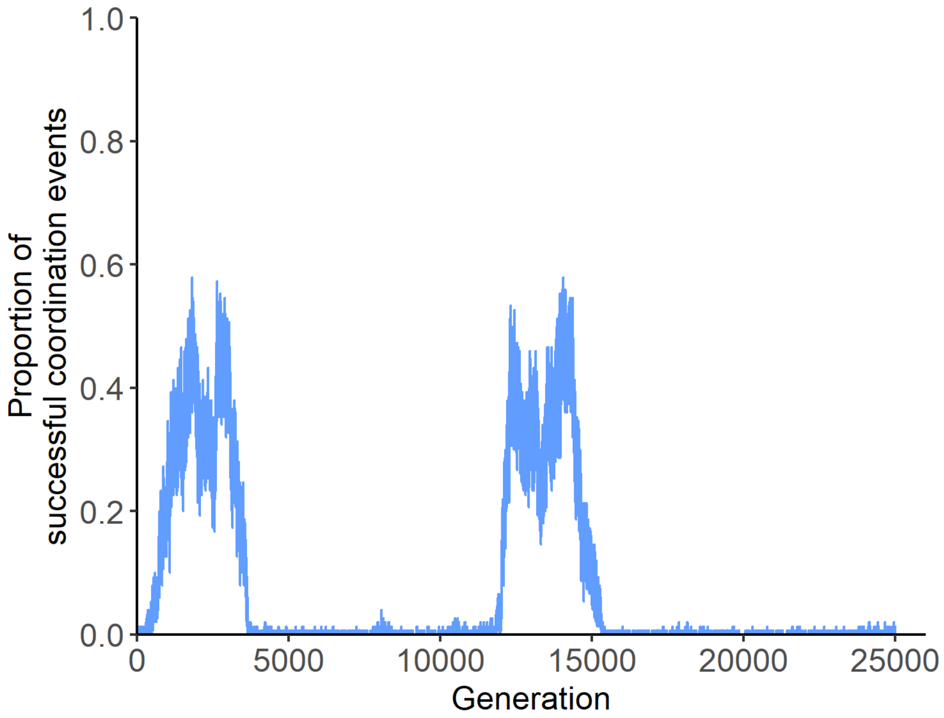

Figure S5: **Proportion of successful interactions over time, for one representative simulation run, when agents can send costless signals.** The y-axis represents the proportion of encounters where both agents choose to Stay. The size of the outside option was $w = .30$, and the initial population played 'Never send a signal, Always Walk'.

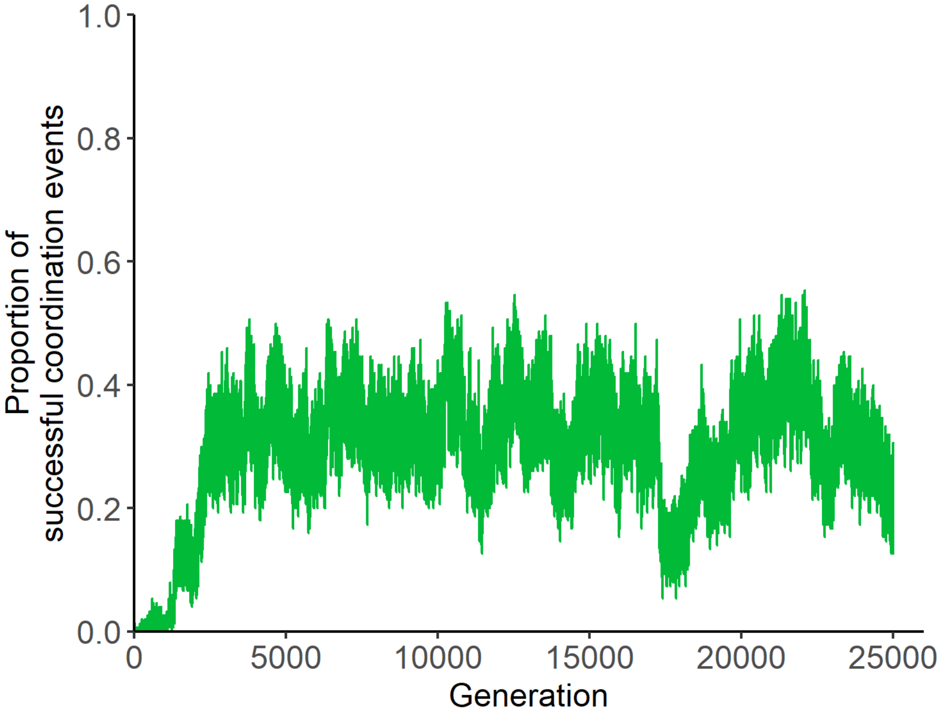

Figure S6: **Proportion of successful interactions over time, for one representative simulation run, when agents can send either a costless or a costly signal.** The y-axis represents the proportion of encounters where both agents choose to Stay. The size of the outside option was $w = .30$, the costly gift had $c = .05$, and the initial population played 'Never send a signal, Always Walk'.

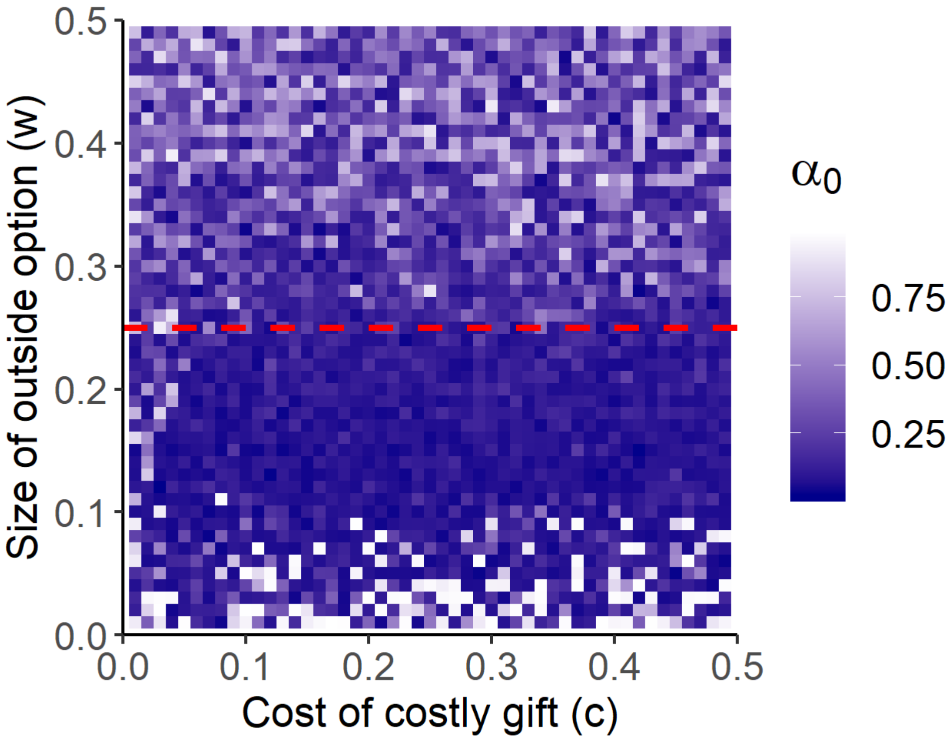

Figure S7: **Costless gift-giving across simulations.** The darkness of a tile is proportional to the prevalence of costless gift-giving in the population in the last 1000 generations of one simulation. Formally, it represents the average value of α_0, the value of S above which an agent sends a costless gift – lower values of α_0 represent more frequent costless gift-giving. Note that c corresponds to the cost of the costly gift, i.e. the *other* gift that agents could send. An agent could send both a costless and a costly gift if it wished to, so a dark tile does not necessarily imply low prevalence of costly gift-giving for that simulation. The red dashed line corresponds to $w = \frac{1}{4}$

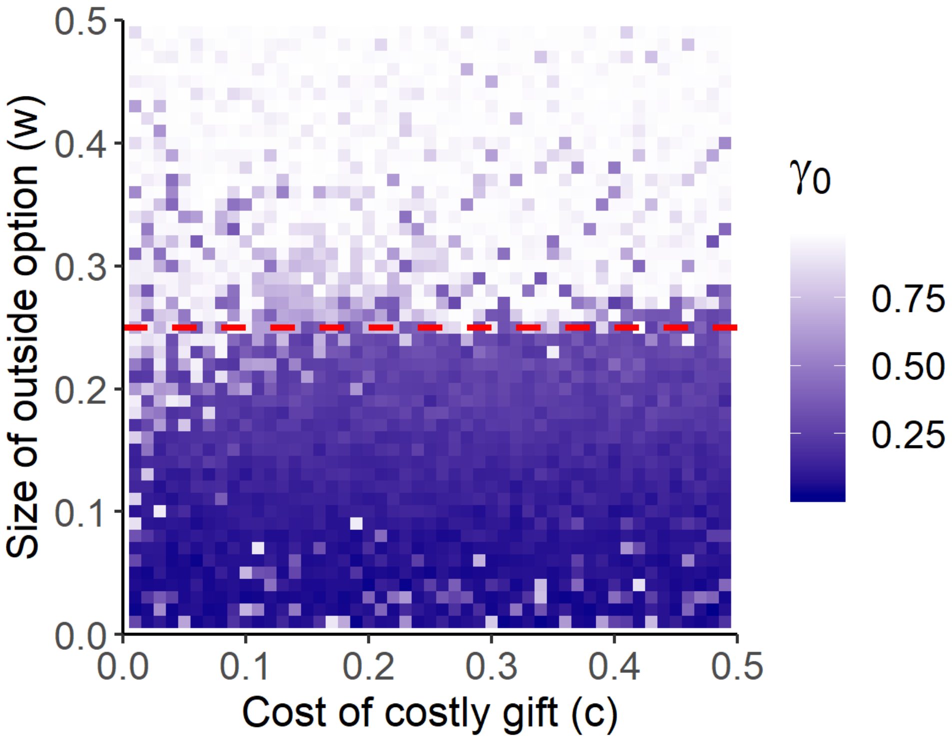

Figure S8: **Trust in costless gifts across simulations.** The darkness of a tile is proportional to the trust agents extend to costless-gift-givers in the population in the last 1000 generations of one simulation. Formally, it represents the average value of γ_0, the value of S above which an agent Stays, given that it has received a costless gift – lower values of γ_0 represent higher likelihood of staying. Note that c corresponds to the cost of the costly gift, i.e. the *other* gift that agents could send. The red dashed line corresponds to $w = \frac{1}{4}$

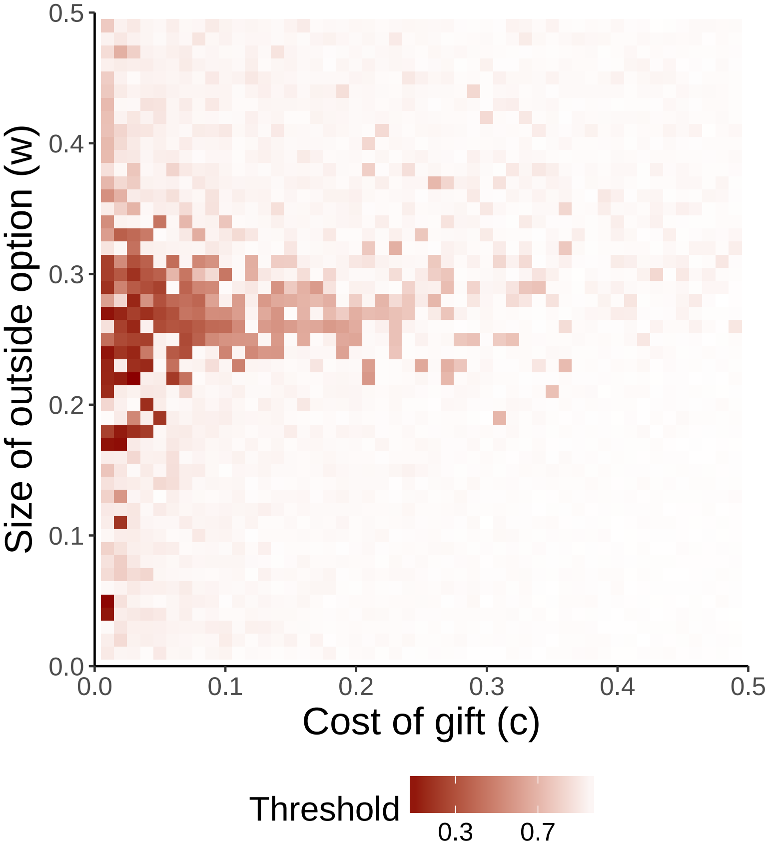

Figure S9: **Costly gift-giving across simulations, for the model where S is drawn from a truncated normal distribution**. The darkness of a tile is proportional to the prevalence of gift-giving in the population in the last 1000 generations of one simulation. Formally, it represents the average value of α_1, the value of S above which an agent sends a costly gift – lower values of α_1 represent more frequent gift-giving.

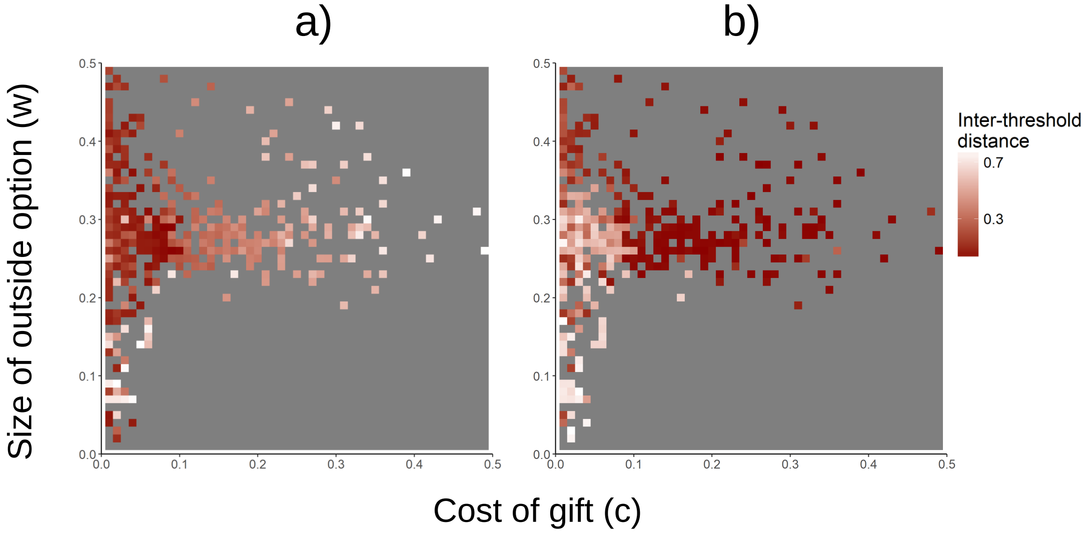

Figure S10: **Prevalence of conditional (a) and unconditional (b) commitment strategies, for the model where S is drawn from a truncated normal distribution**. Colored tiles represent simulations where costly gift-giving evolved ($\alpha_1 < .9$). On panel **a)**, dark tiles represent simulations where conditional commitment strategies have evolved; the brigthness of a tile represents the absolute difference between the threshold value of S above which agents send a costly gift (α_1) and the threshold value of S above which agents Stay if they have received a costly gift (γ_1) – dark values therefore correspond to $\alpha_1 \approx \gamma_1$. On panel **b)**, dark tiles represent simulations where unconditional commitment strategies prevailed; the brightness of a tile represents the absolute difference between the threshold value of S above which agents send a costly gift (α_1) and the threshold value of S above which agents Stay after receiving a costless gift (γ_0) – dark tiles therefore correspond to $\alpha_1 \approx \gamma_0$.

6 References

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