THE UNIVERSITY of EDINBURGH
informatics

ilcc

Institute for Language,
Cognition and Computation

JRC for Logic

Tsinghua University – University of Amsterdam Joint Research Centre for Logic

Causal program induction

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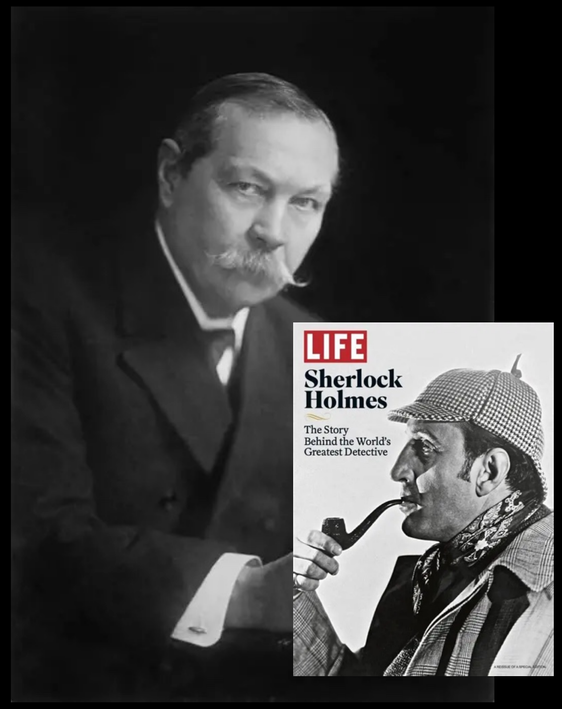
Previously

- SCM, CBN, DAG
- From inference *in* causal models to inference *about* causal models
  - Causal models can get very complex
  - Learning complex models is hard
  - Is it also the case for how the mind learns about causal models?
  - Why do we care about learning causal models?


Kyle of Lochalsh
Mallaig
Inverness
Fort William
Scotland
Aberdeen
Oban
Perth
Dundee
Glasgow
Edinburgh
Cairnryan
Belfast
Stranraer
Carlisle
Newcastle
Isle of Man
Blackpool
Holyhead
Liverpool
Leeds
York
Llandudno
Manchester
Kingston upon Hull
Porthmadog
Crewe
Sheffield


LIFE
Sherlock Holmes
The Story Behind the World's Greatest Detective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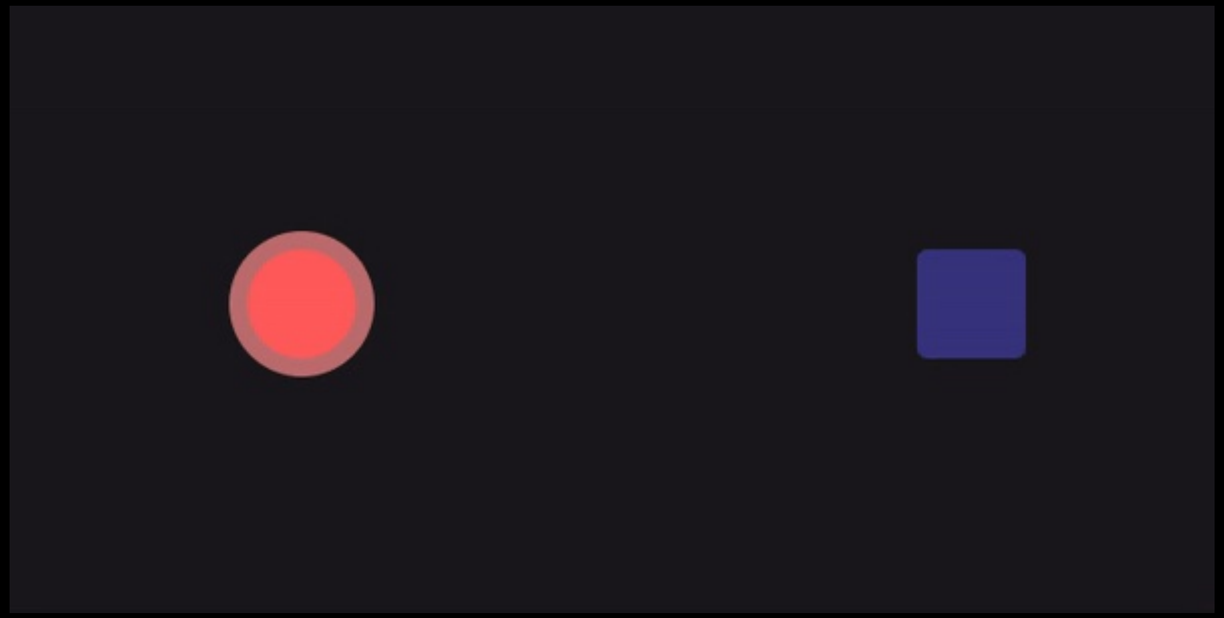

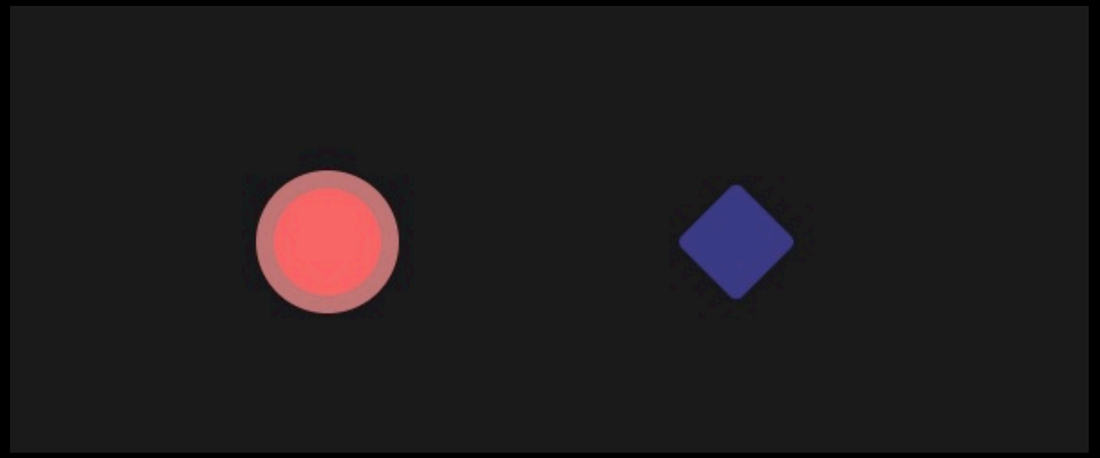

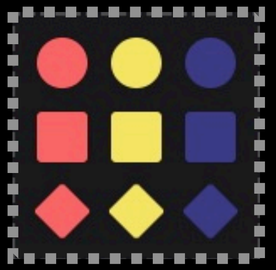

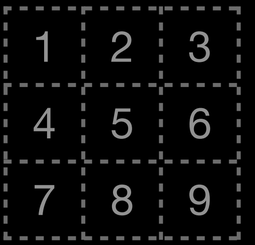
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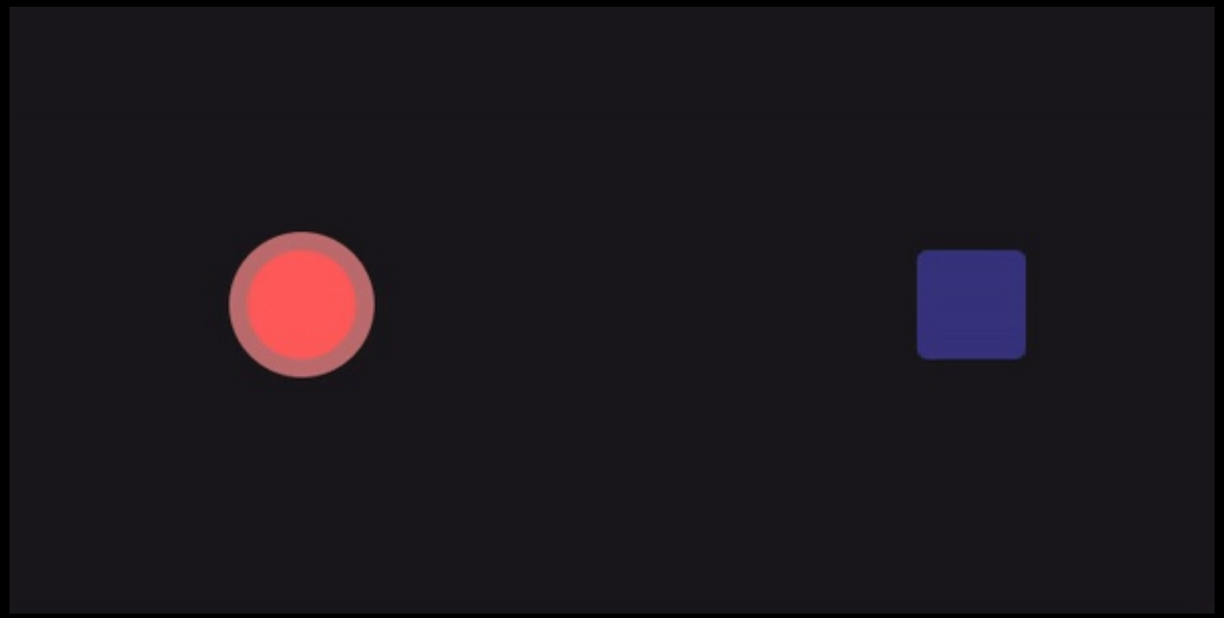

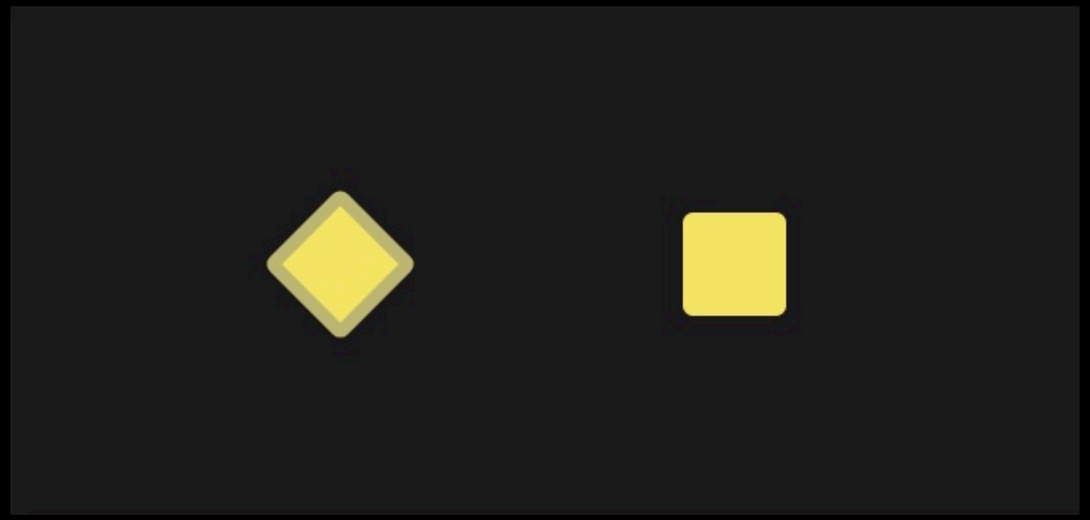

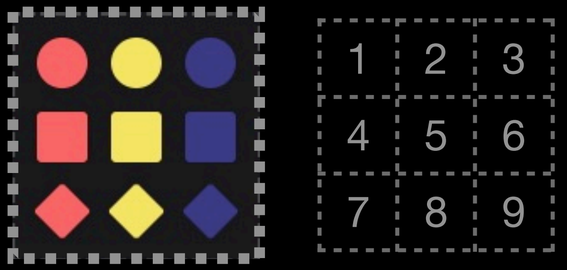
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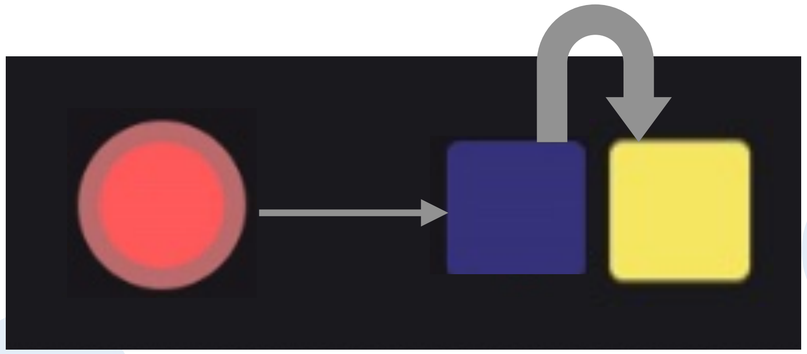
Make things **yellow**?



Red agents make things **change colors**?

Red circles make a **blue square** into **yellow non-circles**?

...

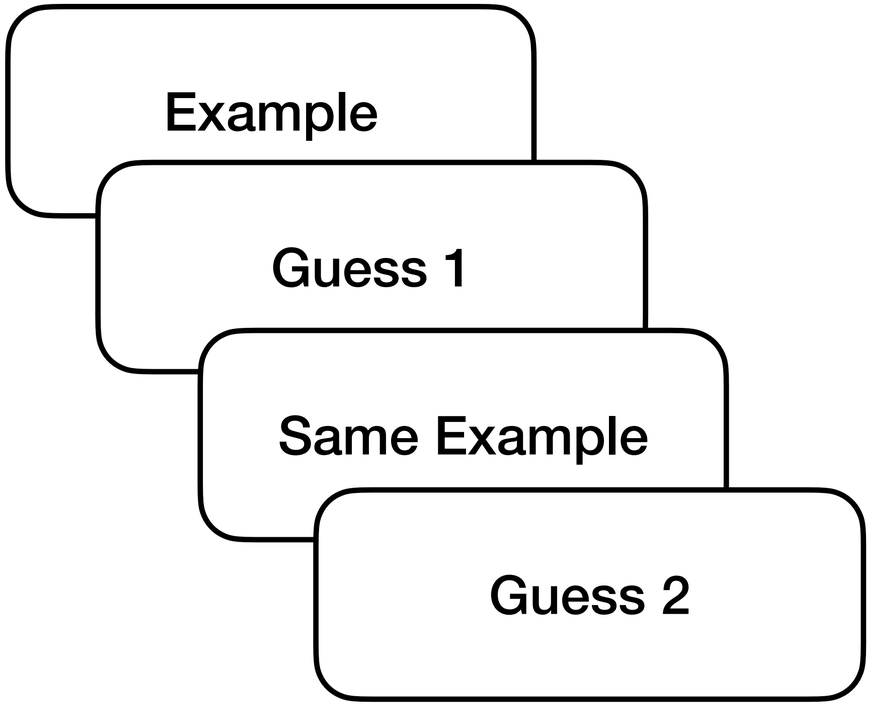
Causal generalization

- People generalize causal relationships systematically, upon seeing very few (even just one!) examples.
- We do this via forming causal concepts that are functional, (often) symbolic, compositional, and probabilistic.

One-shot causal generalization

Zhao et al. (2022)

- Design
  - 6 between-subject example interactions
  - Total n = 120 participants
- Procedure:
  - See 1 causal interaction
  - Make 15 guesses, no feedback



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One-shot causal generalization

Zhao et al. (2022)

One-shot observations

(L1) Normal stone → Magic stone's shape

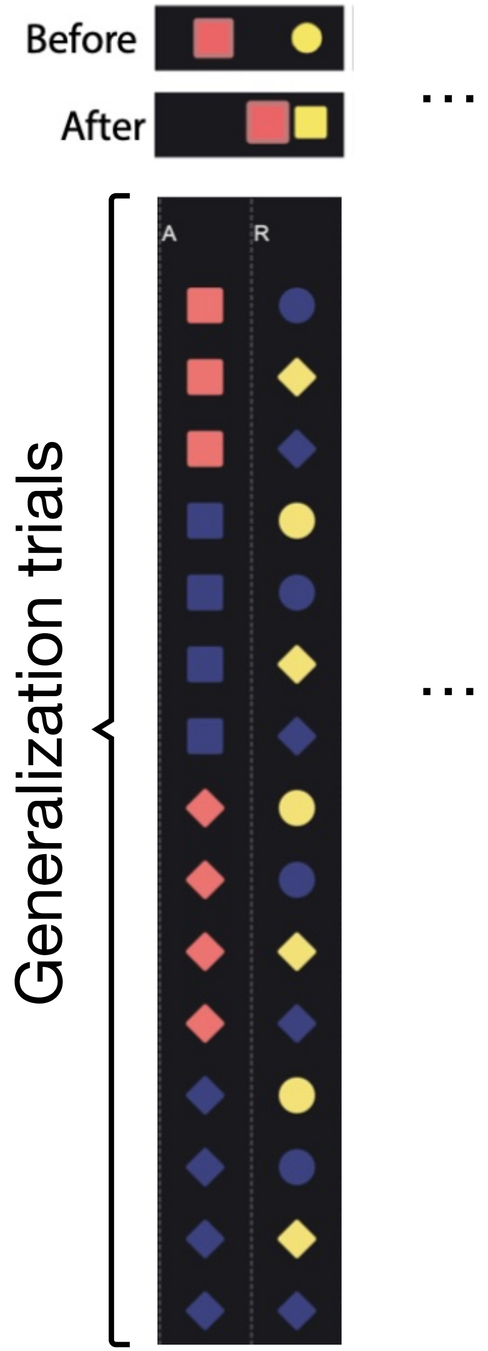
(L2) Normal stone → a new shape

(L3) Normal stone → Magic stone's color

(L4) Normal stone → a new color

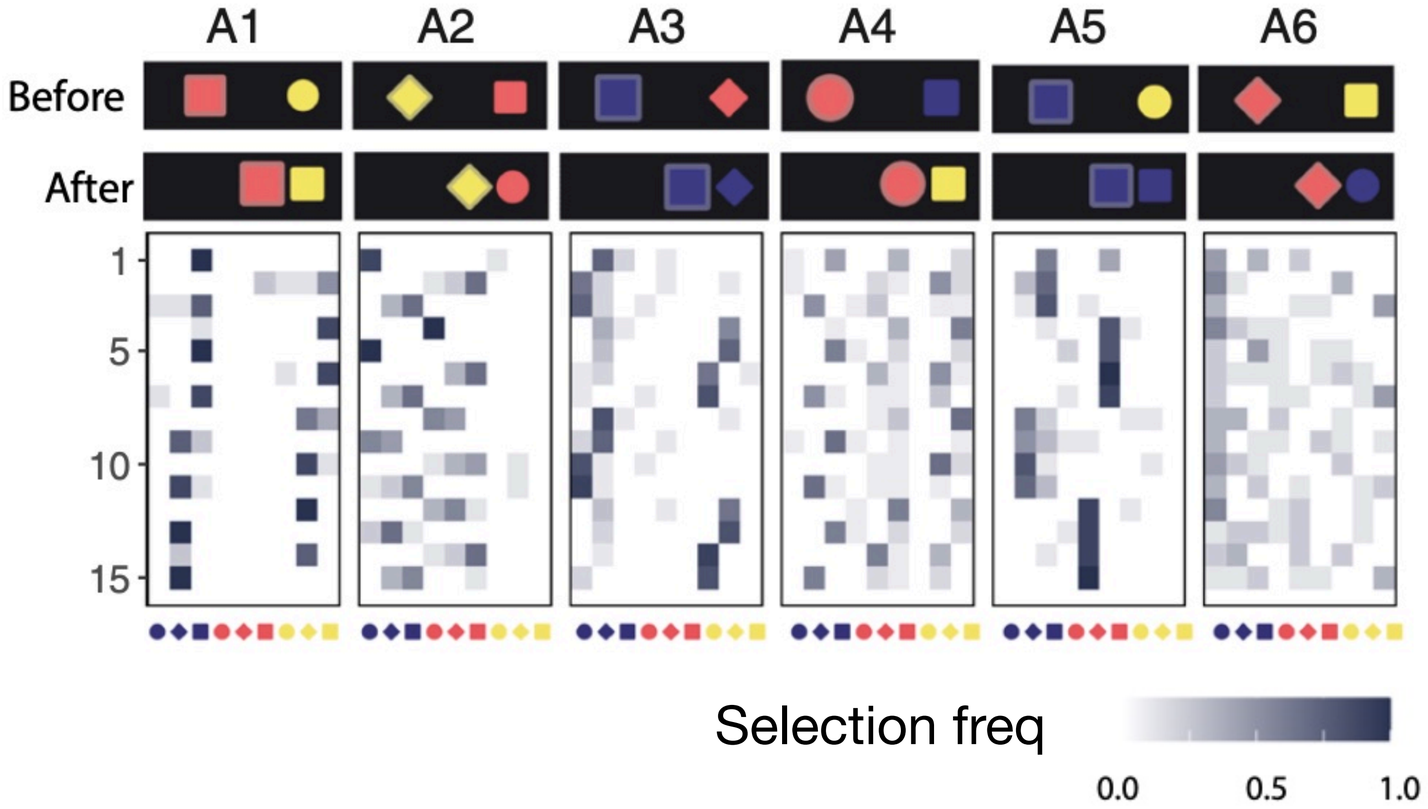
(L5) Normal stone → Magic stone's shape and color

(L6) → a new shape and a new color



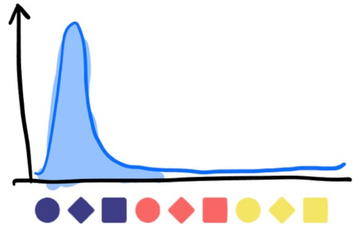
Inter-person generalization similarity

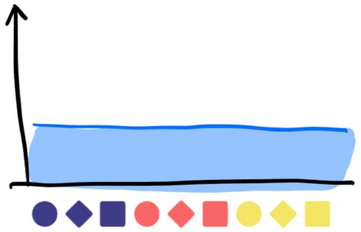
Zhao et al. (2022)

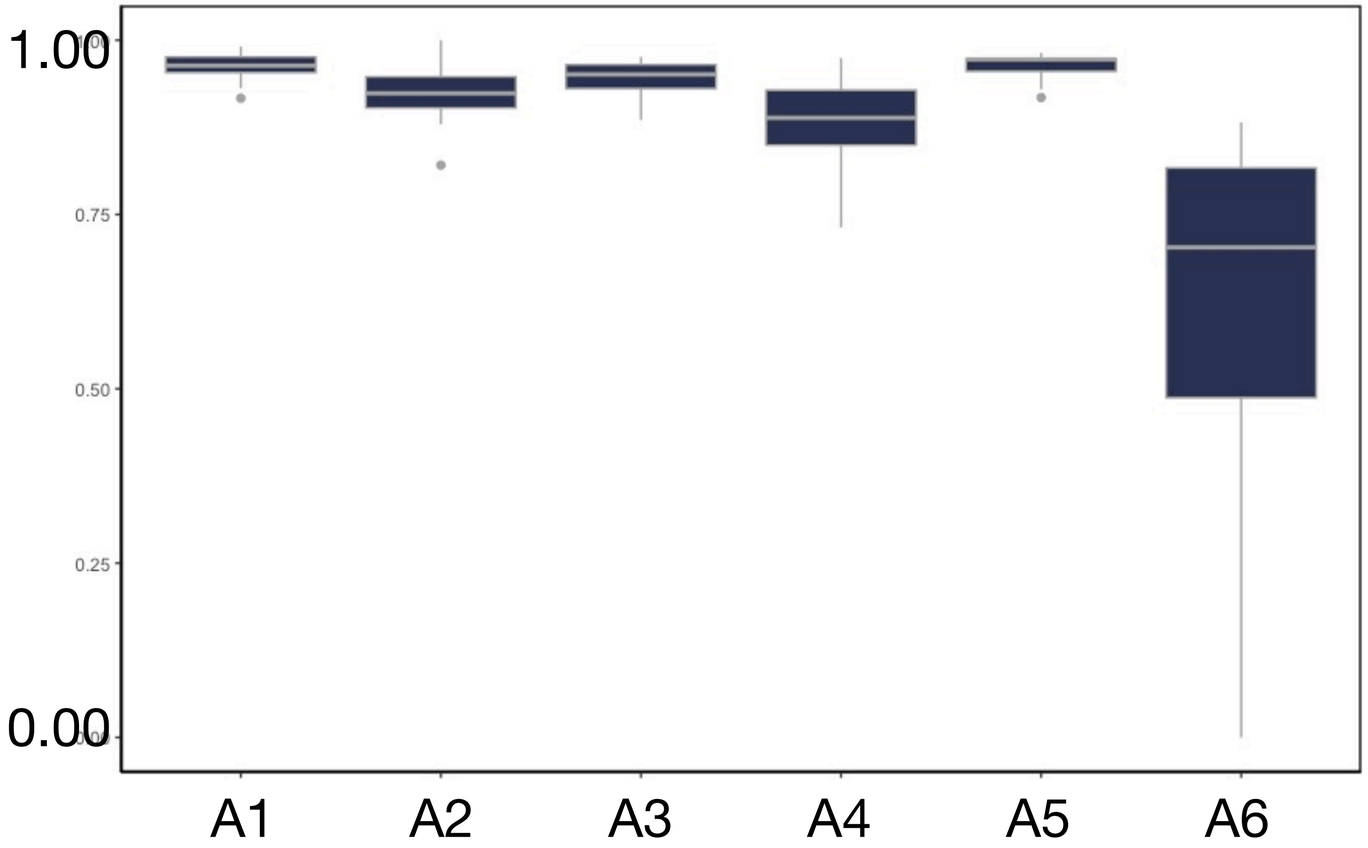


Inter-person generalization similarity

Zhao et al. (2022)







One-shot causal generalization

- People generalize causal relationships systematically, upon seeing very few (even just one!) examples.

- We do this via forming causal concepts that are functional, (often) symbolic, compositional, and probabilistic.

Rule-based concept learning

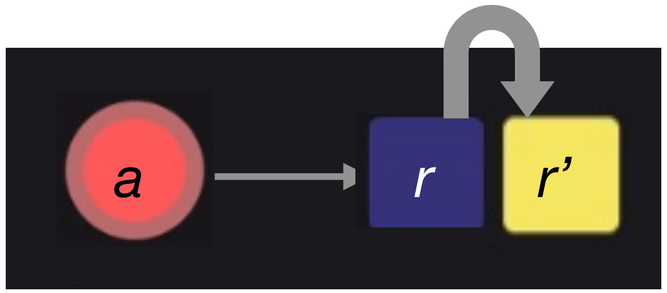
Goodman et al. (2008)

- Concepts are mental representations that are used to discriminate between objects, events, relationships, or other states of affairs.
- Concepts are learned inductively from the sparse and noisy data of an uncertain world.
- Many concepts are formed by combining simpler concepts, and the meanings of complex concepts are derived in systematic ways from the meanings of their constituents.

Handout

Modeling causal concepts

with PCFG (probabilistic context-free grammars)



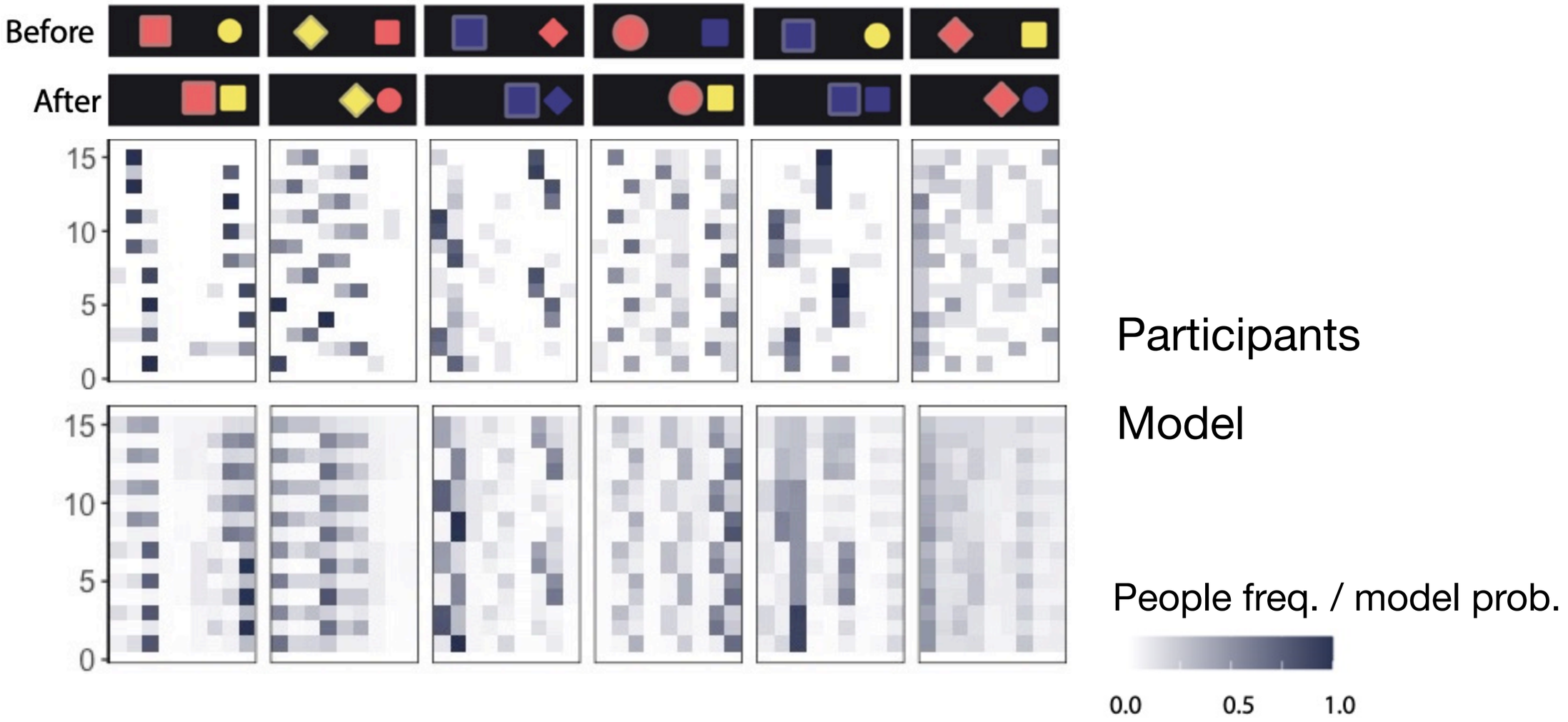
Agent *Recipient* *Result*

color	shape	not
red	circle	and
yellow	square	or
blue	diamond	

Production rules		
Start		
Bind feature	$S \rightarrow \lambda_{\phi_i} : A, \Phi$	
Bind additional	$A \rightarrow B$	$A \rightarrow \text{AND}(B, S)$
Relation	$B \rightarrow \phi_i(r') \Leftarrow C$	$B \rightarrow \phi_i(r') \Leftarrow \neg C$
Reference	$C \rightarrow D$	$C \rightarrow E$
Relative reference	$D \rightarrow \phi_i(a)$	$D \rightarrow \phi_i(r)$
Absolute reference	$E \rightarrow \texttt{value}^{\phi_i}$	

Modeling causal concepts

with PCFG (probabilistic context-free grammars)



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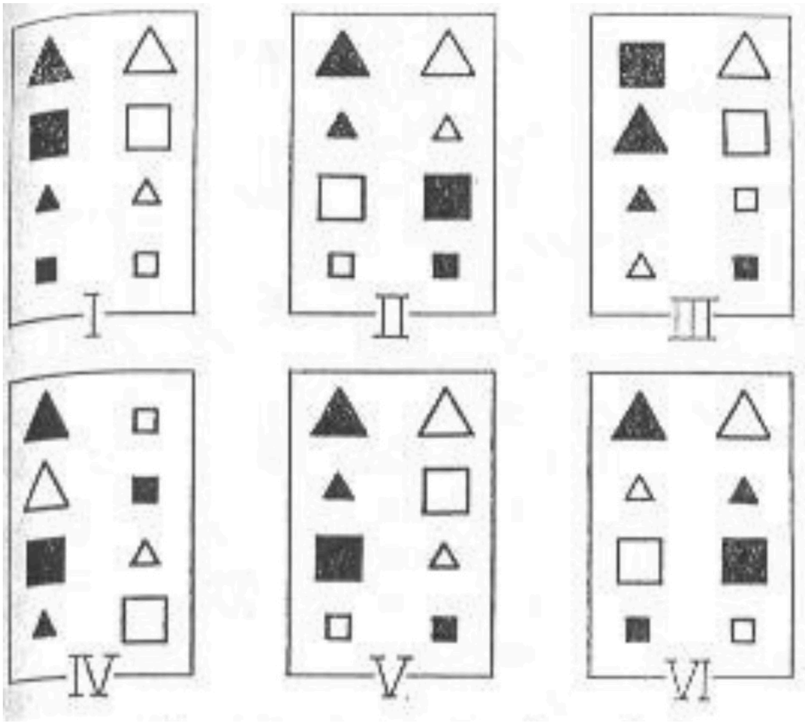
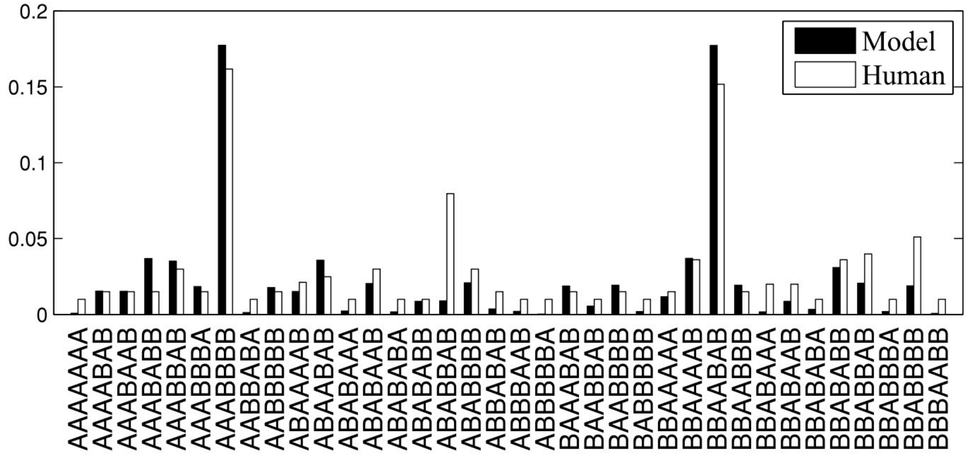
A Rational Analysis of Rule-Based Concept Learning

Noah D. Goodman[a], Joshua B. Tenenbaum[a], Jacob Feldman[b], Thomas L. Griffiths[c]

[a]*Department of Brain and Cognitive Sciences, Massachusetts Institute of Technology*
[b]*Department of Psychology, Rutgers University*
[c]*Department of Psychology, University of California, Berkeley*





Psychological Review

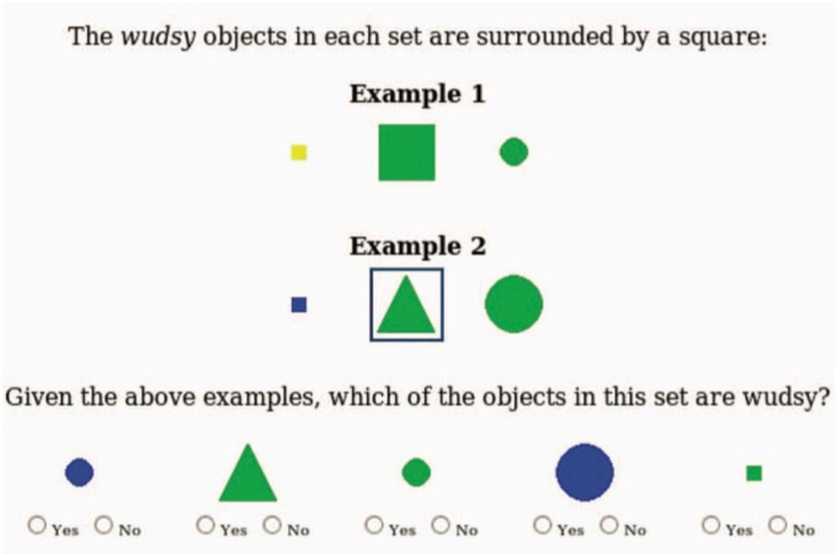
0033-295X/16/$12.00 http://dx.doi.org/10.1037/a0039980

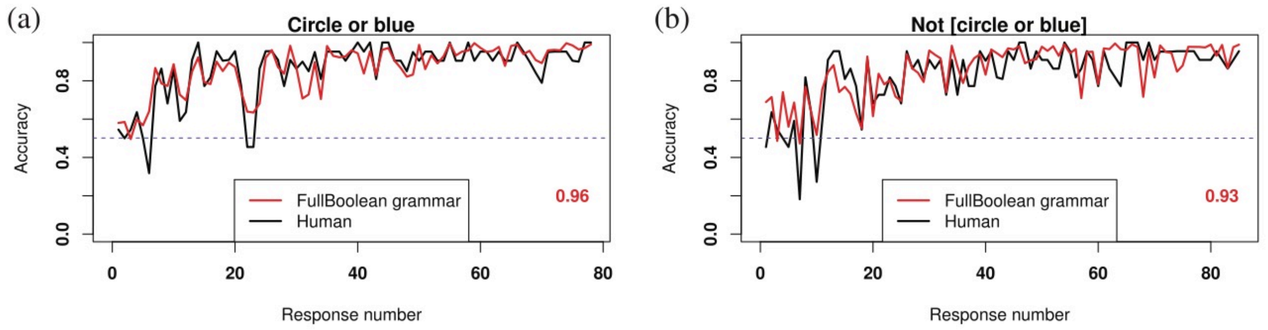
The Logical Primitives of Thought: Empirical Foundations for Compositional Cognitive Models

Steven T. Piantadosi
University of Rochester

Joshua B. Tenenbaum
Massachusetts Institute of Technology

Noah D. Goodman
Stanford University





Contents lists available at ScienceDirect

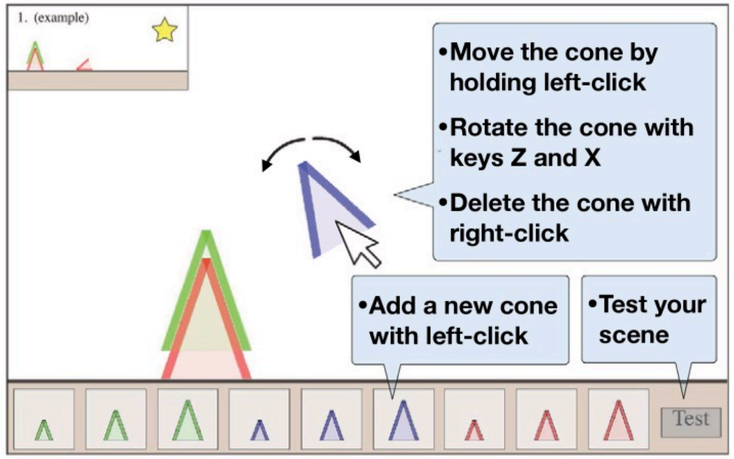
Cognitive Psychology

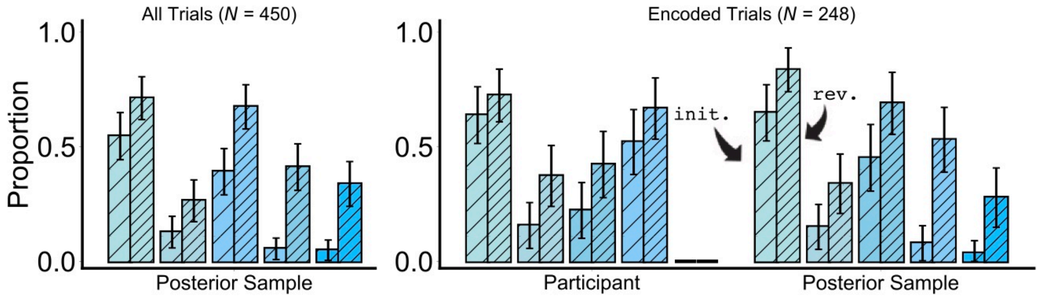
journal homepage: www.elsevier.com/locate/cogpsych

Algorithms of adaptation in inductive inference

Jan-Philipp Fränken [a,*], Nikos C. Theodoropoulos [a,b], Neil R. Bramley [a]

[a] Department of Psychology, University of Edinburgh, United Kingdom
[b] Laboratory of Experimental Psychology, Suor Orsola Benincasa University of Naples, Italy



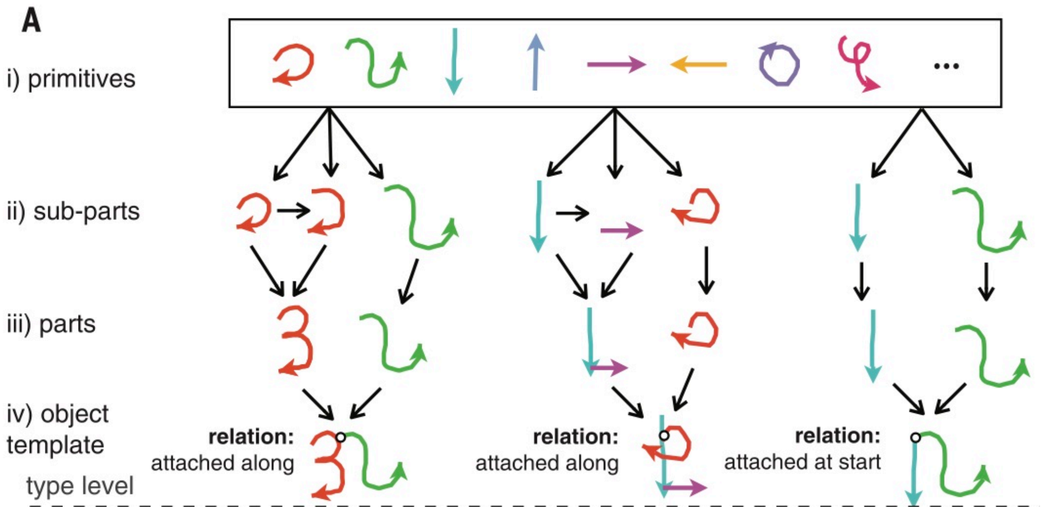


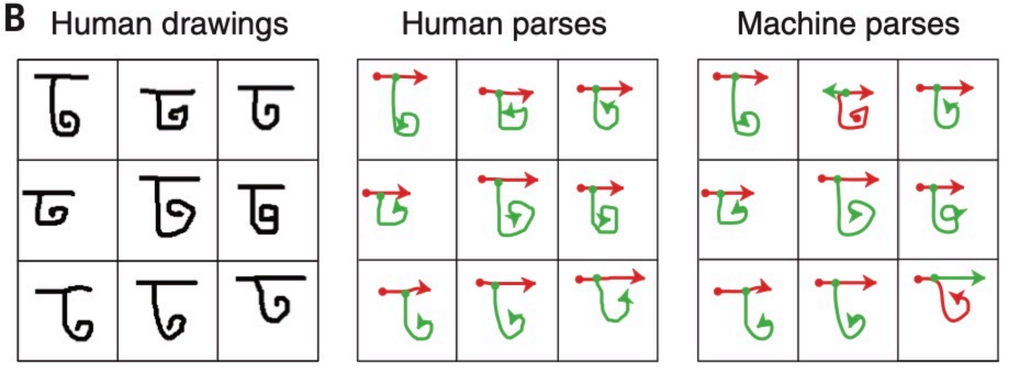
RESEARCH ARTICLES

COGNITIVE SCIENCE

Human-level concept learning through probabilistic program induction

Brenden M. Lake,[1]* Ruslan Salakhutdinov,[2] Joshua B. Tenenbaum[3]





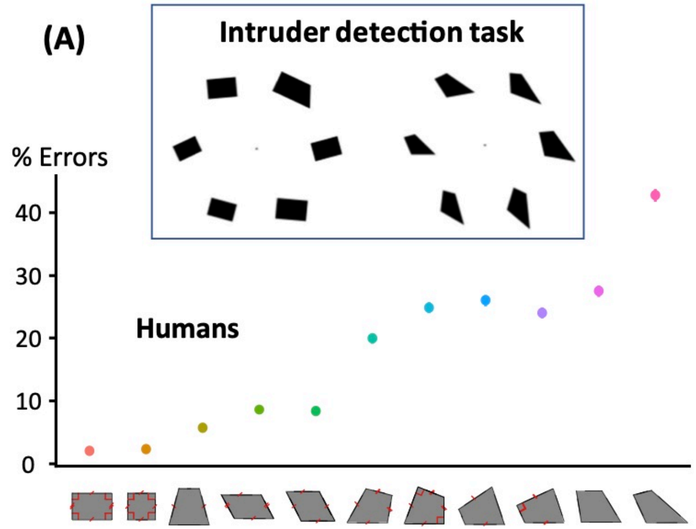
Trends in
Cognitive Sciences

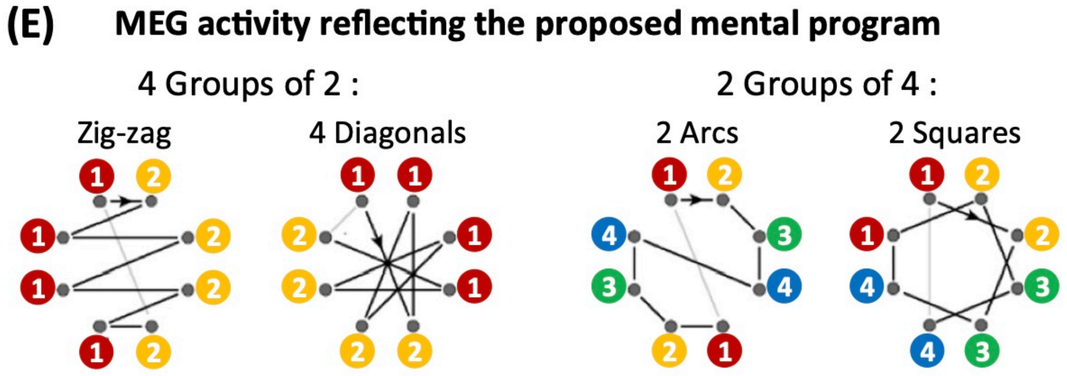
CellPress

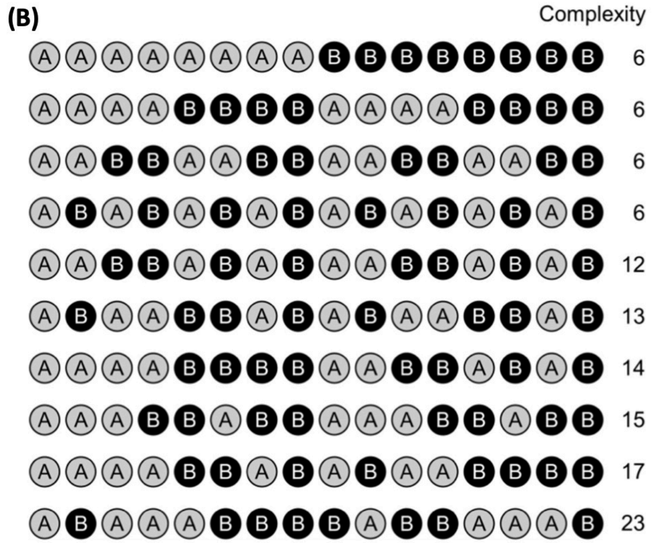
Opinion

Symbols and mental programs: a hypothesis about human singularity

Stanislas Dehaene, [1,2,*] Fosca Al Roumi, [1] Yair Lakretz, [1] Samuel Planton, [1] and Mathias Sablé-Meyer [1]







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Opinion

Origins of numbers: a shared language-of-thought for arithmetic and geometry?

Stanislas Dehaene[1,2], Mathias Sablé-Meyer[1,2], and Lorenzo Ciccione[1,2,*]

Language primitive	General meaning	Role in geometry	Role in arithmetic
`Int: =`		Integers are needed for the Repeat operation	Classical definition of **Peano arithmetic**
`| One`	Number 1		
`| Next(Int)`	Successor function		
`Program: =`			
`| Repeat([Int=2]){Program}`	Repetition: Repeat a program a certain number of times	Repeat any line or shape, e.g. /// or ○○○	• **Set** of *n* items • Set of sets = **Multiplication**
`| Program; Program`	Concatenation: run one program, then another	Join two shapes, e.g. —○	**Union** of two sets = **Addition**
`| Subprogram{Program}`	Nesting: call any subprogram, then resume the original program	Draw a shape using another shape, e.g. square of circles ○○○ ○○○ ○○○	**Algebraic structure** recursively combining + and × **Square**, **Factors**, etc.

Summary

- Treating causal models as concepts
- Using PCFG to assign priors over a structured, open-ended concept space
- Example studies
- Tomorrow: how to do inference when the normalization term is intractable?